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THE UNIVERSITY OF ALBERTA

BEHAVIOR OF CANTILEVERED FLAT PLATE SLABS

by

MAYNARD MACLENNAN VOLLAN

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

EDMONTON, ALBERTA

NOVEMBER 1964

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "BEHAVIOR OF CANTILEVERED FLAT PLATE SLABS submitted by Maynard Maclennan Vollan in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

This analytical investigation deals with the behavior of flat plate cantilevered slabs resulting from changes in some of the variables. Those considered are the amount of overhang, the column bending stiffness, the perimeter beam bending and torsional stiffness, the loading, and Poisson's ratio.

The structure analysed was a four panel square plate, with equal overhangs on all four sides, supported on nine non-settling columns. Perimeter beams stiffen the slab edges.

An elastic numerical method based on Newmark's plate analogy was used to solve the problem. It revealed that positive bending moments, in the same order of magnitude as the maximum interior positive bending moments, can exist near the slab edges when the perimeter beams are large.

ACKNOWLEDGMENTS

This thesis was prepared under the direction of Dr. S.H. Simmonds, Associate Professor of Civil Engineering. The author wishes to express his appreciation for the advice received during the progress of this investigation. Mr. C. Rolfson assisted in the preparation of the computer programs used.

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CHAPTER I

INTRODUCTION

1.1 Introductory Remarks

The constant attempt by architects to achieve new effects sometimes creates problems for structural engineers. One of these is the increasingly popular practice of insetting the exterior columns of slabs in order to achieve exterior faces free from supporting structural elements. At present the behavior of the cantilevered portions of these slabs is only vaguely understood. If such structures are to be safely and economically constructed it is imperative that the effects of the many variables involved be considered in the design. This investigation was restricted to elastic flat plate cantilevered slabs.

1.2 Object

The object of this study was to determine the behavior of flat plate cantilevered slabs resulting from changes in some of the variables thought to be important. Specifically, the effects on the deflections and bending moments of the following variables were investigated: the ratio of the length of overhang to the distance between the columns, the bending and torsional stiffnesses of the edge beams, the bending stiffness of the supporting columns, the loading, and Poisson's ratio.

1.3 Scope

The cantilevered slab investigated consisted of four square panels with equal overhangs on all four sides, as shown in FIGURE 1. The slab

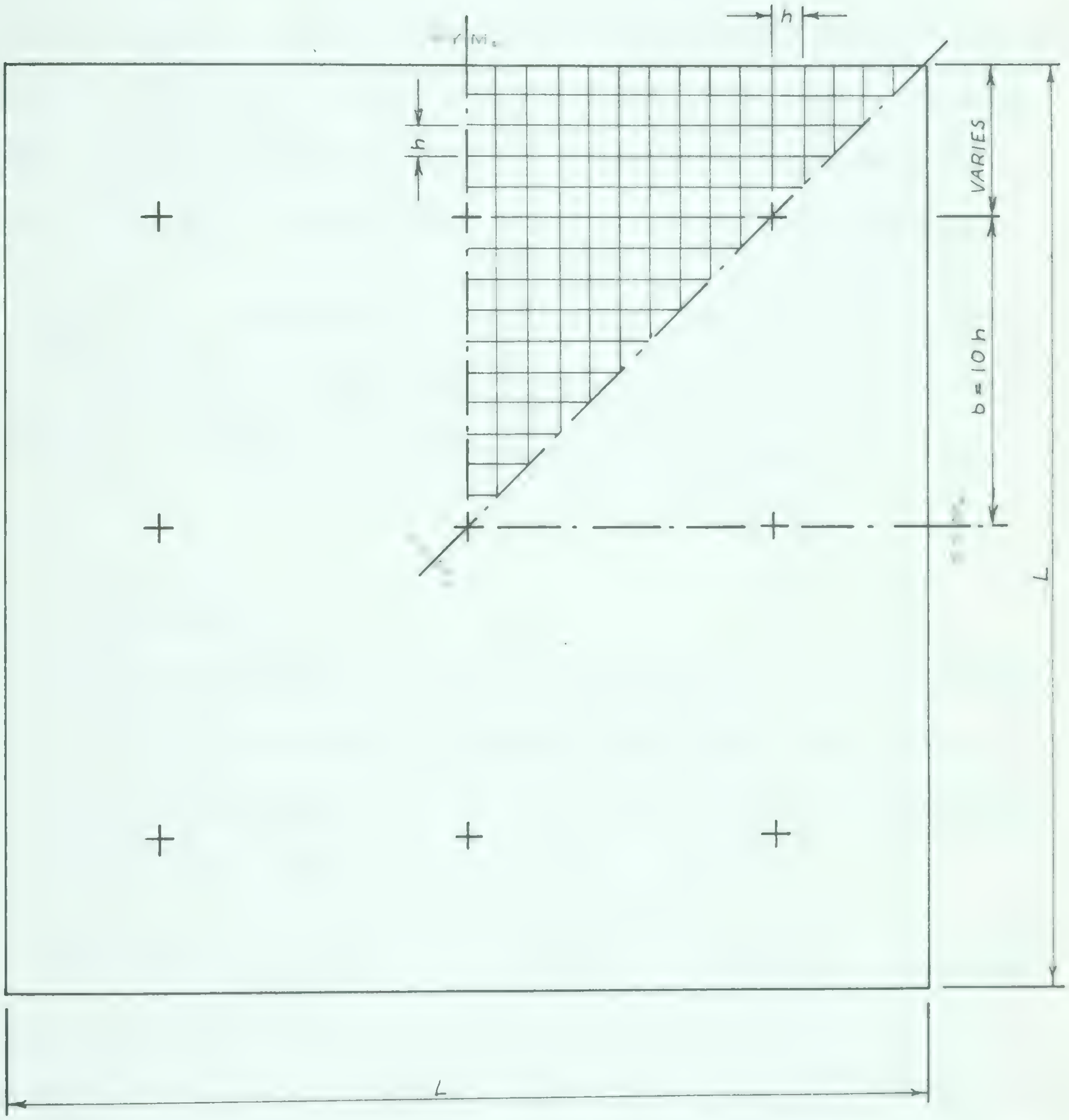


FIGURE 1 - ARRANGMENT OF SLAB INVESTIGATED

was made symmetrical about the centerlines and diagonals in order that a solution could be obtained by considering only one eighth of the system. The panels were supported on nine non-settling columns, each having the same bending stiffness in both directions. Elastic beams with both bending and torsional stiffnesses strengthened the edges of the slab. Although it is quite common to achieve additional support from beams spanning between the columns, this investigation was limited to flat plate action.

The elastic deformations and bending moments at a large number of points on the slab were determined for the 124 combinations of variables examined. The edge beam bending and twisting moments were also investigated.

1.4 Historical Review

Considerable analytical and experimental work has been carried out in the past on the behavior of concrete structural slabs. Despite this extensive work, there exists a void in the literature on the behavior and design of cantilevered slabs.

In 1961, Simmonds (1) did some work on a one panel square slab with equal overhangs on all four sides. In some of the seven solutions obtained he found that for a uniformly distributed load, the bending moment at the outside face of the columns produced compression on the top of the slab. This present study was initiated, in part, to examine the magnitude and extent of this behavior.

At present there is no set method of designing cantilevered slabs. The deflection pattern and the variation in the bending moments across the

width of the cantilevered portion are usually based on the judgment of the engineer, a practice that does not always assure a satisfactory structure.

1.6 Notation

b = span length of one panel, center to center distance between columns

C = a measure of the tortional rigidity of a beam cross-section

E_b = modulus of elasticity of the beam material

E_c = modulus of elasticity of the column material

E_s = modulus of elasticity of the slab material

G = $\frac{E_b}{2(1+\mu)}$ shear modulus of elasticity of the beam material

h = distance between joints or node points of the grid

H = $\frac{E_bI_b}{N_b}$ ratio of the beam flexural stiffness to slab stiffness

H' = $\frac{E_bI_b}{Nh} = \frac{b}{h} H$ = a measure of the flexural stiffness of a beam

I_b = moment of inertia of the cross-section of a beam

I_c = moment of inertia of the cross-section of a column

I_s = $\frac{t^3}{12}$ moment of inertia per unit width of slab

J = $\frac{CG}{Nb}$ ratio of beam torsional stiffness to plate stiffness

J' = $\frac{CG}{Nh} = \frac{b}{h} J$ a measure of the torsional stiffness of a beam

k = $\frac{4E_cI_c}{L_c}$ stiffness of column fixed at far end

K = $\frac{k}{N}$ a measure of the total flexural stiffness of a column

L_c = effective length of column

m_x = $-N(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2})$ bending moment about X-X axis per unit width of slab

m_y = $-N(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2})$ bending moment about Y-Y axis per unit width of slab

m_{xy} = $-N(1-\mu)\frac{\partial^2 w}{\partial x \partial y}$ twisting moment per unit width of slab

m = bending moments or twisting moment acting on a section of slab of width h

M = $-E_b I_b \frac{\partial^2 w}{\partial x^2}$ flexural bending moment in beam parallel to Y-Y axis

M_c = $-k\frac{\partial w}{\partial x}$ end moment applied to column about Y-Y axis

N = $\frac{E_s I_s}{(1-\mu^2)}$ measure on slab stiffness

OH = distance between slab edge and column divided by b

p = uniformly distribued load per unit of length

q = uniformly distributed load per unit of area

t = thickness of slab

T = $-CG\frac{\partial}{\partial x}(\frac{\partial w}{\partial y})$ torsional moment in a beam

w = vertical deflection of the slab, positive downward

x,y,z = rectangular reference coordinates

μ = Poisson's ratio

B=b Used only on Figures

Q=q Used only on Figures

CHAPTER II

METHOD OF ANALYSIS

2.1 Introductory Remarks

Generally the analysis of structural slabs involves the solving of the governing differential equation for a particular set of loading and boundary conditions. However, when internal discontinuities or elastic boundary conditions are encountered it may be impossible to write the differential equations. Since this problem is encountered with slabs supported on flexible columns and beams some other method of analysis must be used.

Newmark proposed a mathematical analogue (3)(4)(5)(6) in which the slab is approximated by a grid of rigid weightless bars connected by elastic springs. Considering the equilibrium of these bars gives rise to algebric equations in terms of the deflections which are identical to those obtained from the finite difference equivalents of the differential equation. With the analogue, however, modifications are possible to permit the consideration of any type of boundary conditions, such as elastic columns and beams. Using this method the analysis of a problem consists of writing the equilibrium equations at every grid point on the analogue and solving them simultaneously for the deflections. The stress resultants may be obtained from these deflections by using the relations in APPENDIX A.

2.2 Basic Assumptions of Analogue

The mathematical analogue or model consists of bars connected

by flexible elastic springs, with torsion springs connecting adjacent parallel bars. FIGURE 2 shows a diagramatic representation of this analogue near an exterior corner. The following assumptions are necessary in addition to those usually made for theelastic behavior of slabs (2)(8).

(a) The bars are weightless and rigid.

(b) The mass of the bars and the external loads are concentrated at the elastic hinges.

(c) The resultants of direct stresses are bending moments at the elastic hinges and at the ends of each bar.

(d) The resultant of vertical shears are shearing forces acting at the elastic hinges and at the ends of each bar.

(e) The resultants of the horizontal shearing stresses are twisting moments concentrated in the torsion springs.

The added assumptions required to account for the intregal effects of the beams and columns with the slab are:

(f) The behavior of the beams in both bending and torsion is linearly elastic.

(g) The neutral axes of the beams coincide with the middle plane of the slab.

(h) The forces and moments acting on a beam are distributed along a line and not over a finite width.

(i) Torsion in the beams is uniform between node points and the effects of warping are neglected.

(h) The action of the column is concentrated at a point, located at the intersection of the column center line and the middle plane of the slab.

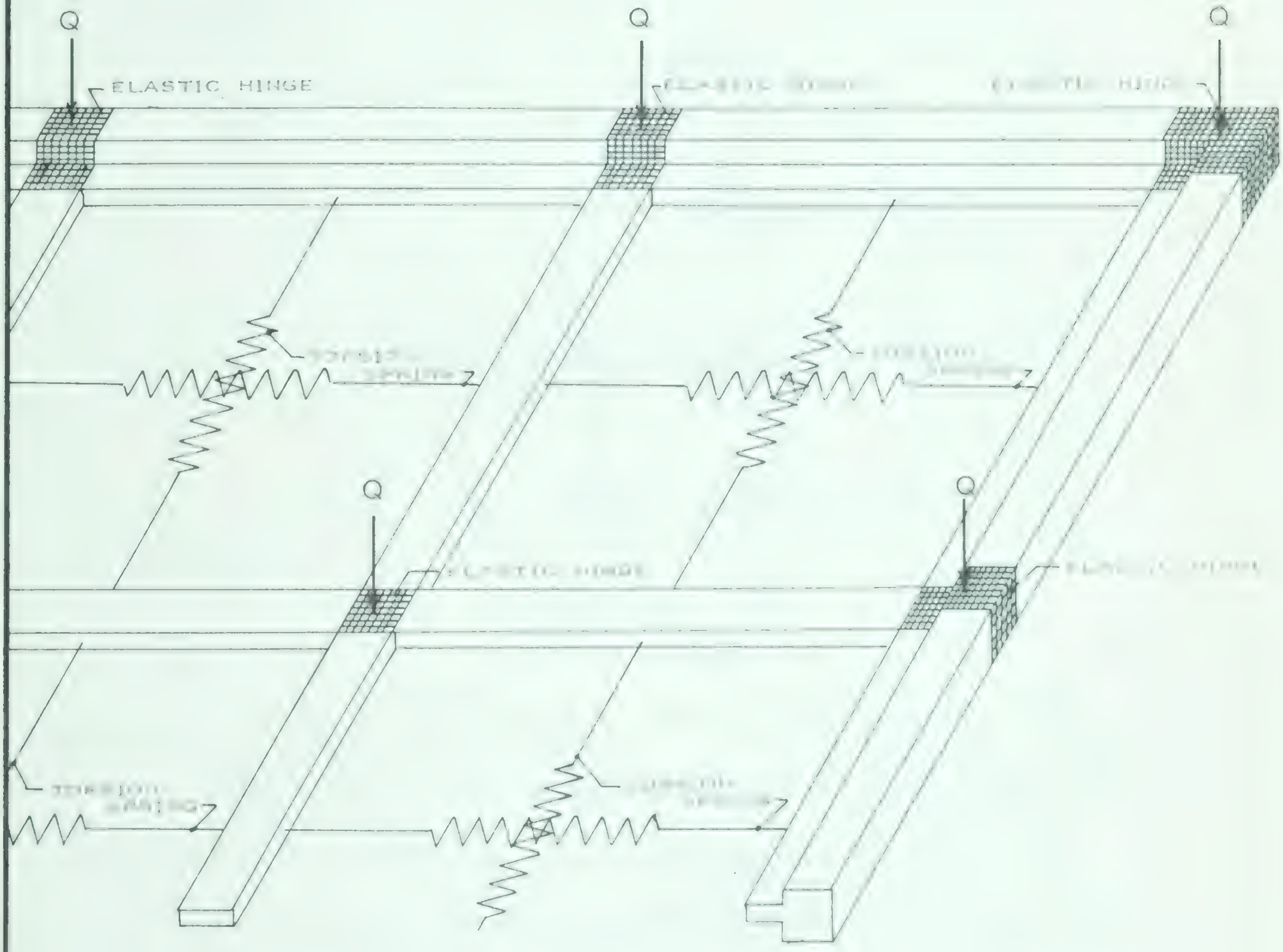
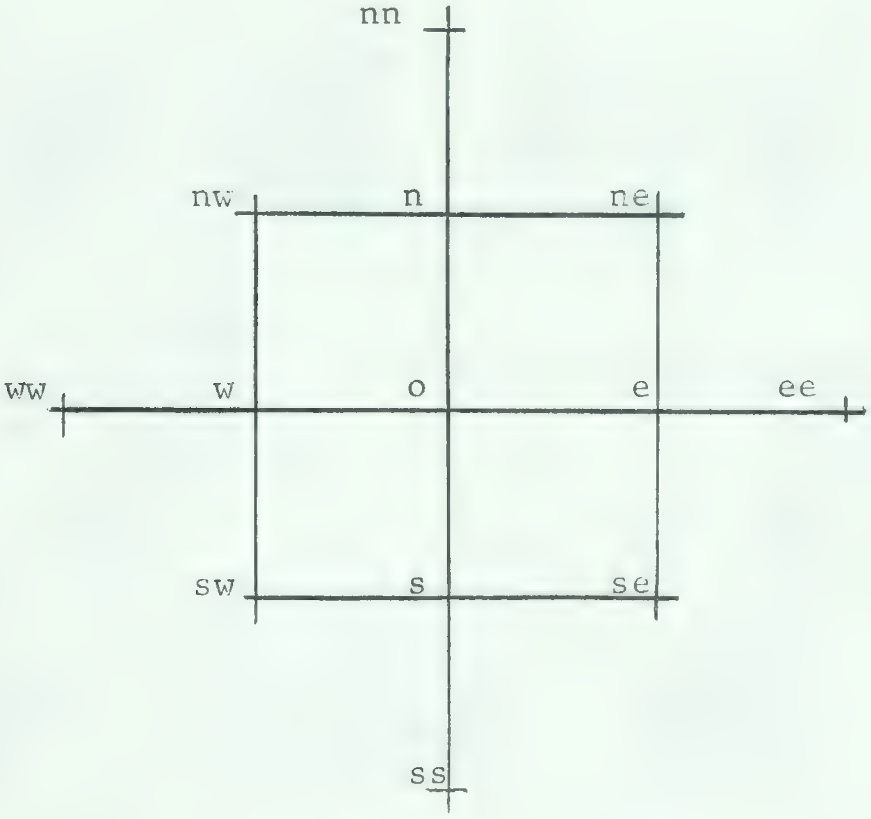


FIGURE 2 - NEWMARK'S PLATE ANALOGUE FOR AN EXTERIOR CORNER

2.3 Derivation of Analogy Operator

The equations controlling the deflections of the analogue have been derived elsewhere (3)(4)(5). The following example illustrates the method which was employed.

The points on the analogue are referred to by the designation shown below.



The analogue shown in FIGURE 2 represents an exterior corner of a slab which has edge beams but no corner column. The forces and moments developed in the elastic springs and hinges are shown in FIGURE 3. Equilibrium of the corner node (FIGURE 4) results in the following equation.

$$Q - V_{os} - \bar{v}_{os} - V_{oe} - \bar{v}_{oe} = 0 \tag{1}$$

These shears are expressed in terms of the bending and twisting moments by considering the equilibrium of the bars.

$$V_{os} = \frac{1}{h} (M_{os} - M_{so}) \tag{2}$$

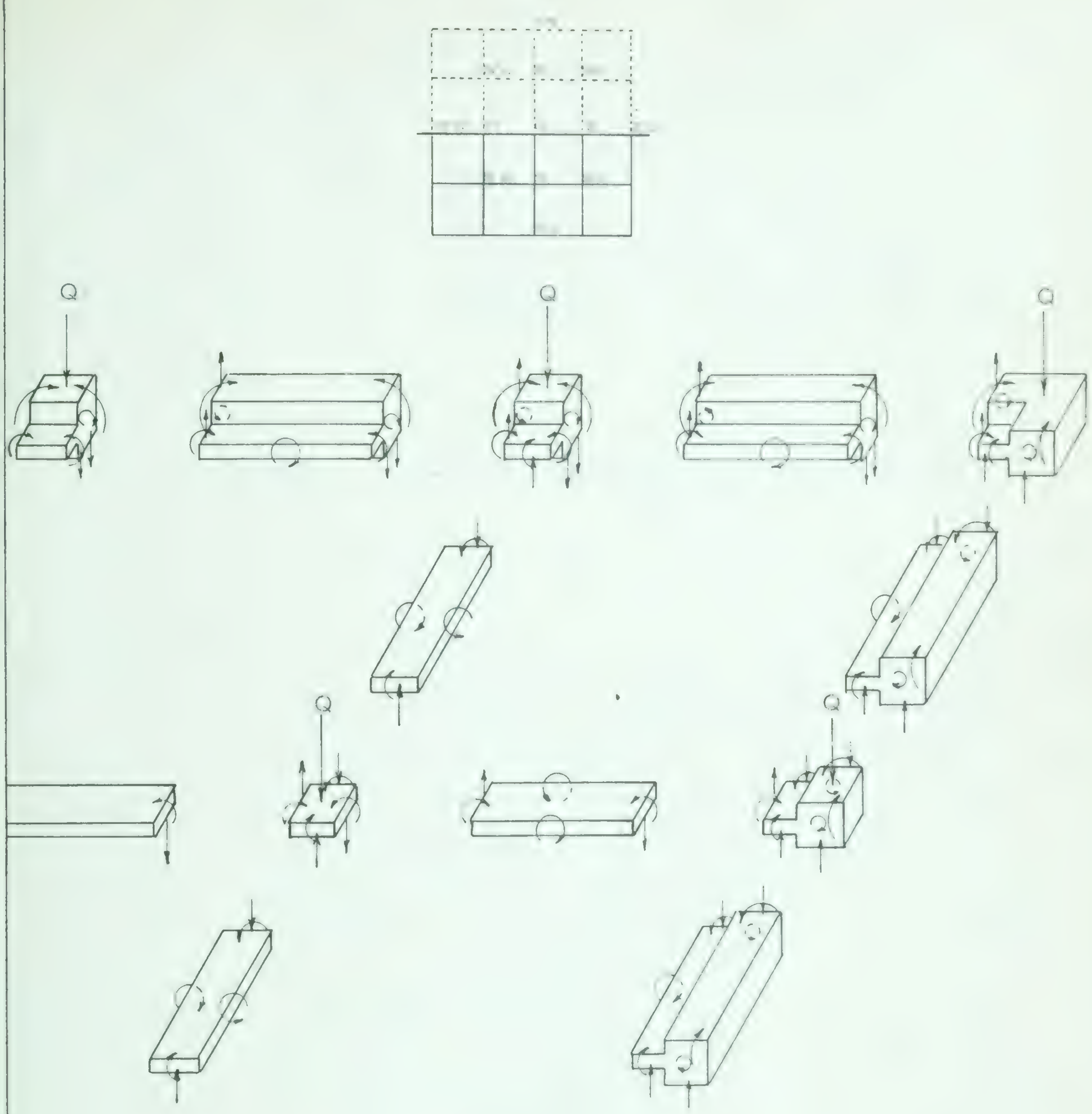


FIGURE 3 – FORCES ACTING ON PLATE ANALOGUE OF AN EXTERIOR CORNER

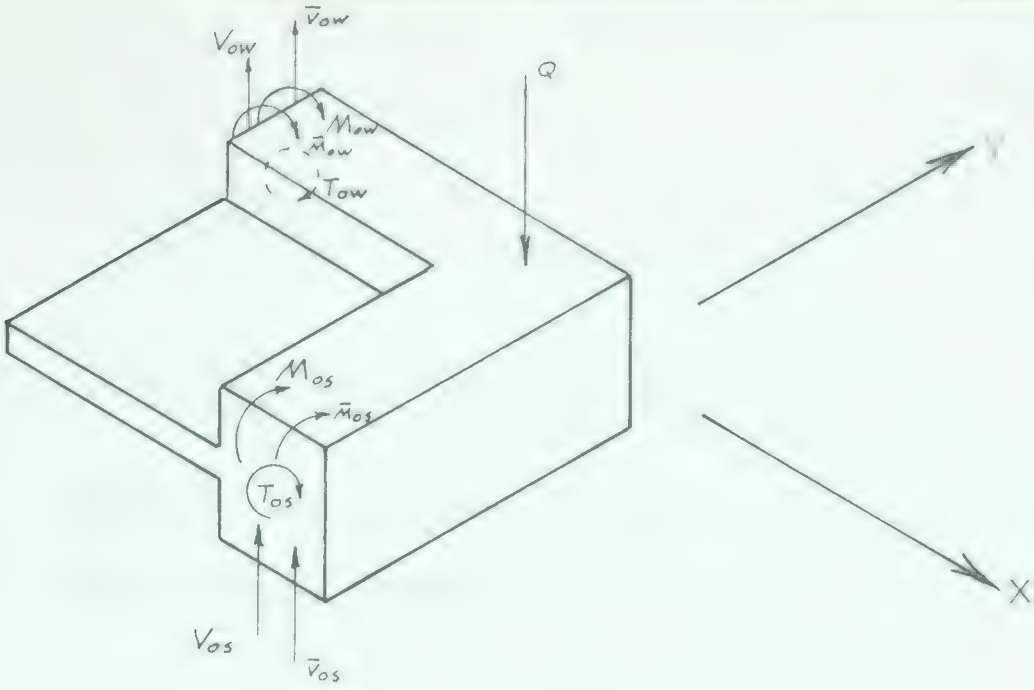


FIGURE 4 - ANALOGUE MOMENTS FOR CORNER NODE

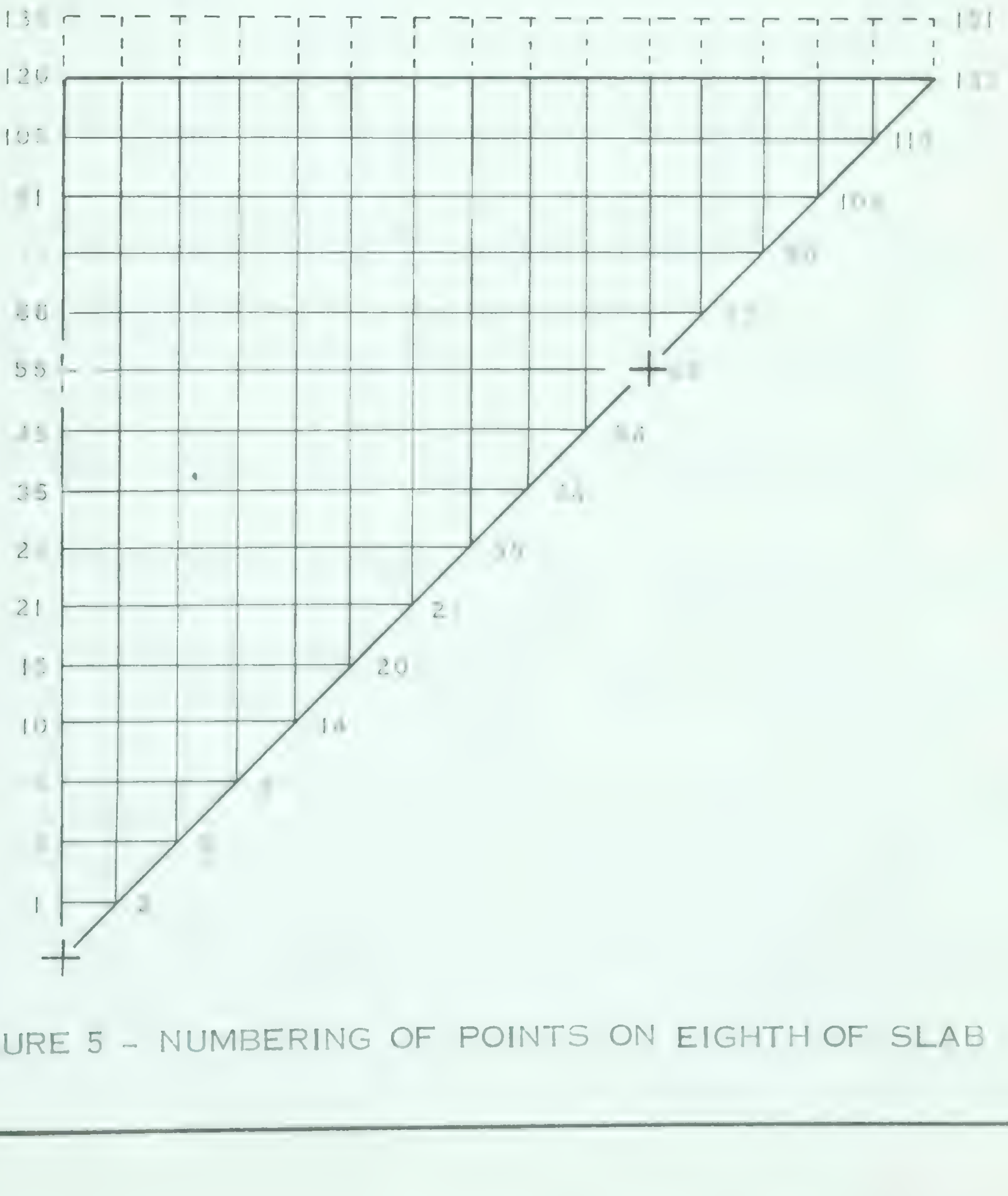


FIGURE 5 - NUMBERING OF POINTS ON EIGHTH OF SLAB

$$V_{oe} = \frac{1}{h}\,(M_{oe} - M_{eo}) \tag{2}$$

$$\overline{v}_{os} = \frac{1}{h}\,(\overline{m}_{os} - \overline{m}_{so} - \overline{m}^{C}{}_{xy})$$

$$\overline{v}_{oe} = \frac{1}{h}\,(\overline{m}_{oe} - \overline{m}_{eo} - \overline{m}^{C}{}_{xy})$$

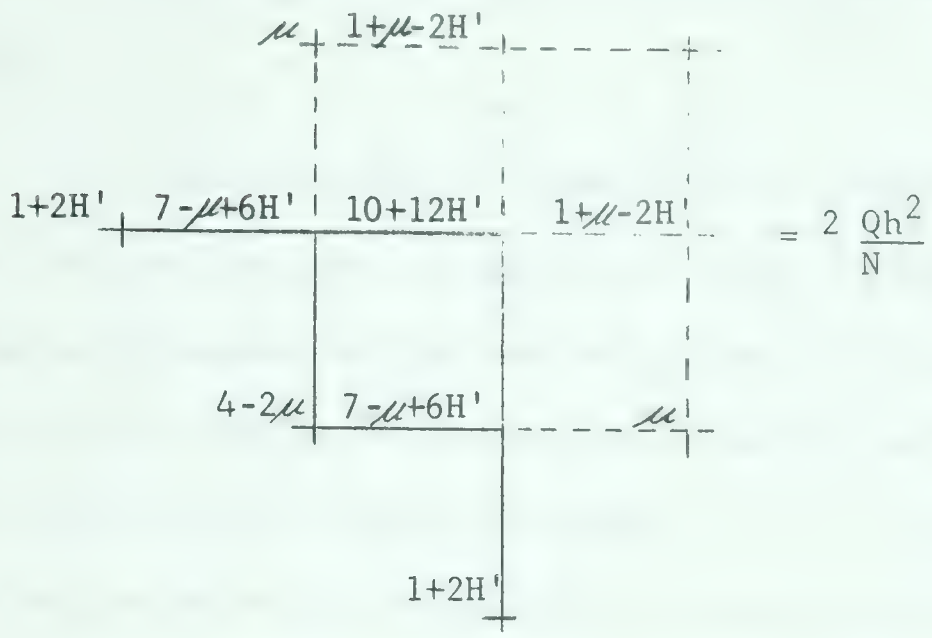
These moments can be expressed in terms of the deflections by the relationships contained in APPENDIX A as follows:

$$\begin{aligned}
M_{os} &= \frac{NH'}{h}\,(-w_{\overline{n}} - \mu w_{w} + 2w_{o} + 2\mu w_{o} - \mu w_{\overline{e}} - w_{s})\\
M_{so} &= \frac{NH'}{h}\,(-w_{o} - \mu w_{sw} + 2w_{s} + 2\mu w_{s} - \mu w_{\overline{\overline{se}}} - w_{ss})\\
M_{ow} &= \frac{NH'}{h}\,(-\mu w_{\overline{n}} - w_{w} + 2w_{o} + 2\mu w_{o} - w_{\overline{e}} - \mu w_{s})\\
M_{wo} &= \frac{NH'}{h}\,(-\mu w_{\overline{\overline{nw}}} - w_{ww} \quad 2w_{\overline{w}} + 2\mu w_{w} - w_{o} - \mu w_{sw})\\
\overline{m}_{os} &= \frac{N}{2h}\,(-w_{\overline{n}} - \mu w_{w} + 2w_{o} + 2\mu w_{o} - \mu w_{\overline{e}} - w_{s})\\
\overline{m}_{so} &= \frac{N}{2h}\,(-w_{o} - \mu w_{sw} + 2w_{s} + 2\mu w_{s} - \mu w_{\overline{\overline{se}}} - w_{ss})\\
\overline{m}_{ow} &= \frac{N}{2h}\,(-\mu w_{\overline{n}} - w_{w} + 2w_{o} + 2\mu w_{o} - w_{\overline{e}} - \mu w_{s})\\
\overline{m}_{wo} &= \frac{N}{2h}\,(-\mu w_{\overline{\overline{nw}}} - w_{ww} + 2w_{w} \quad 2\mu w_{w} - w_{o} - \mu w_{sw})\\
\overline{m}^{C}{}_{xy} &= \frac{N}{h}\,(w_{w} - \mu w_{w} - w_{o} + \mu w_{o} - w_{sw} + \mu w_{sw} + w_{s} - \mu w_{s})
\end{aligned} \tag{3}$$

Substituting equations (2) and (3) into (1) results in the following equation:

$$\begin{aligned}
&\mu w_{\overline{\overline{ne}}} - (1 + \mu + 2H')w_{\overline{n}} + (1 + 2H')w_{\overline{\overline{ee}}} - (7 - \mu + 6H')w_{\overline{e}}\\
&+ (10 + 12H')w_{o} - (1 + \mu - 2H')w_{w} + (4 - 2\mu)w_{\overline{\overline{se}}} - (7 - \mu + 6H')w_{s} + \mu w_{sw}\\
&+ (1 + 2H')w_{ss} = 2Qh^{2}/N
\end{aligned}$$

This can be expressed in operator notation as



Note that the operator involves ficticious deflections $w_{\overline{ne}}$ $w_{\bar{n}}$, $w_{\bar{w}}$, and $w_{\overline{sw}}$. Equations for the solution of $w_{\bar{n}}$ and $w_{\bar{w}}$ are obtained from the moment equations of equilibrium about the X and Y axis at point o. Similarily $w_{\overline{ne}}$ and $w_{\overline{sw}}$ are found from the equilibrium equations at points w and s respectively.

All generated imaginary points can be eliminated in this manner or by considering the continuity of the slab. For example the equation $w_{\bar{n}} - w_s = w_n - w_{\bar{s}}$ which equates the slab slopes on both sides on an internal discontinuity, is frequently used for this purpose.

2.4 Solving Simultaneous Linear Algebraic Equations

The major problem in solving problems using a numerical or finite difference approach is obtaining the correct solution to the large number of simultaneous equations generated. The use of digital computers has greatly reduced the time required for this purpose but some estimate of the accuracy obtained should be made. Since computers carry only a predetermined number of digits, there may be a loss in significance when nearly identical numbers are subtracted and when truncations or round

offs occur. This is not usually a serious problem with finite difference equations.

One characteristic of Newmarks plate analogue is that the equilibrium equations for any particular point include only points nearby while the coefficients of the remaining terms in the equations are zero. If the resulting matrix is solved using a general method, much efficiency is lost in dealing with the zero elements. It is possible to overcome much of this by writing the equations so that all the non-zero elements fall on vectors parallel to the main diagonal. White (7) did this for a complete rectangular plate in which thirteen continuous vectors run parallel to the main diagonal. When eight fold symmetry is made use of in a square plate, again there are thirteen diagonal vectors but eight of these run continuously for only short distances. These broken paths follow systematic patterns similiar to those shown in FIGURE 6 when the equations are written successively for each point as numbered in FIGURE 5. Since all the other elements in the matrix are zero, a 151 x 151 matrix can be read in with only 1,963 numbers rather than the usual 22,801. The computer is programmed to place the numbers in the proper locations.

Further advantage can be realized if the equations are now solved by the Gauss Elimination Method which White (7) recommends for finite difference equations. This method affects only elements between the extreme vectors above and below the main diagonal thus eliminating computer time usually wasted outside this range. The Gauss Elimination Method also permits the equations for several different loading conditions to be solved simultaneously.

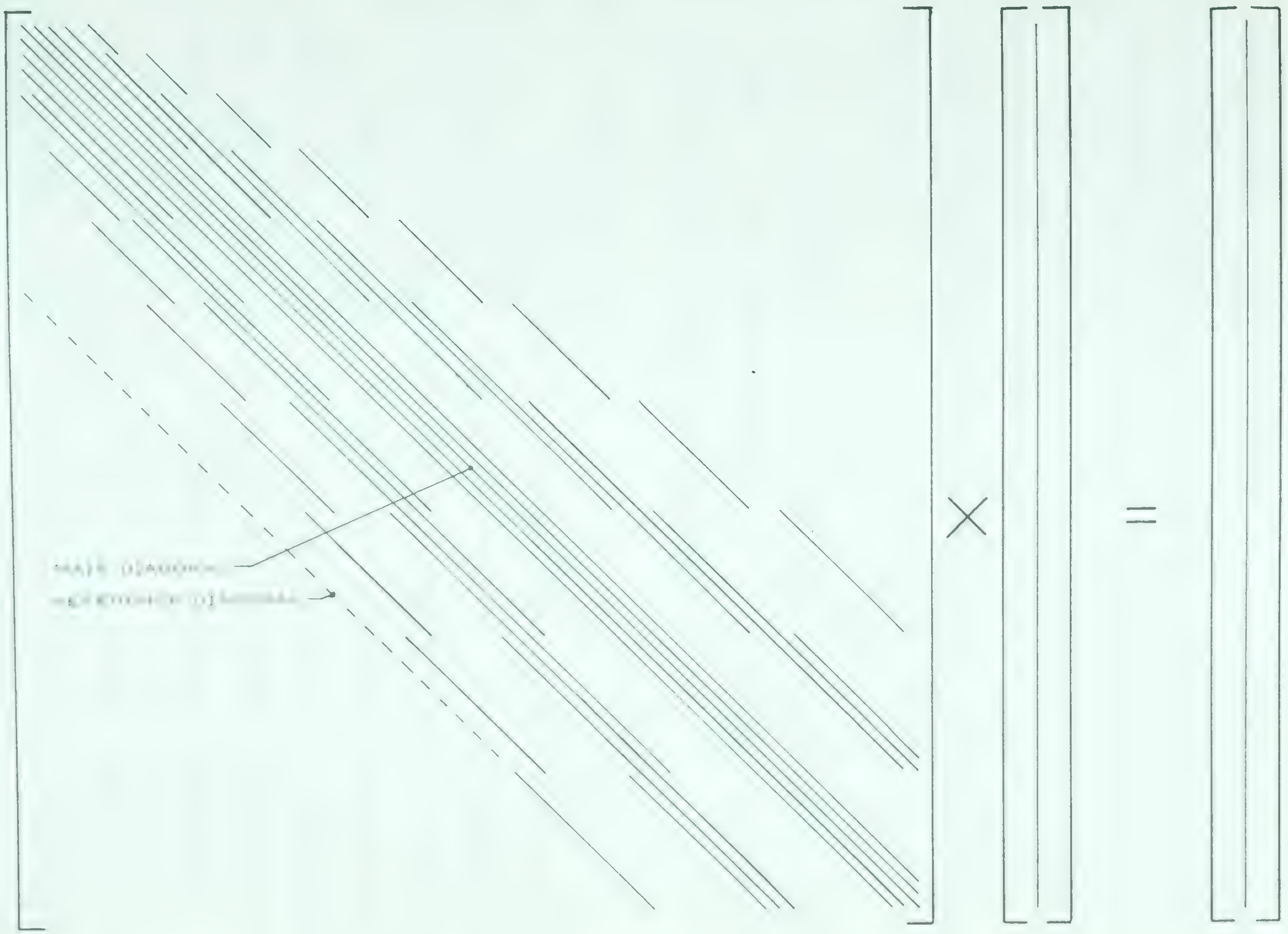


FIGURE 6 - PLATE ANALOGUE MATRIX FORM FOR SQUARE PLATE WITH EIGHT FOLD SYMMETRY

"Diagonal subscritping", as White (4) calls it, permits a considerable saving in the amount of computer memory space required. Although this involves calling up the rows of the matrix in the usual manner, the columns run parallel to the main diagonal instead of vertically. The extreme left vector is extended the full height of the matrix and is used as an origin. Using this reference system, a set of 151 equations generated from a plate with eight fold symmetry can be solved in only two fifths of the memory space usually required. This saving increases as more equations are solved.

A description of this procedure along with the required computer program is contained in APPENDIX C. The technique is further illustrated by the solution of a small sample problem.

2.5 Accuracy of Solutions

Since the equation from the plate analog is identical to the corresponding finite difference expressions, some preliminary work was carried out with finite difference on beam problems. It was found that for a uniformly loaded simple beam the deflections obtained with five subdivisions are within 3.3% of the accepted values and the bending moments are exact. Increasing the number of subdivisions to twenty increases the deflection accuracy to 0.37% and again the moments are exact.

The solution to one set of simultaneous equations was obtained carrying eight and then sixteen significant digits. The deflections and bending moments resulting from the two solutions agreed to at least four significant figures.

If a section is taken through the cantilevered portion, the sum of the bending moments across the section must satisfy statics. For each solution this check was made on from three to six sections depending on the overhang ratio. Although the maximum error revealed was 1.34%, the average error was only in the order of 0.01%.

The bending moments may be somewhat in error immediately adjacent to the columns. Since the columns are assumed to act at points, purely elastic solutions result in infinite stresses around the columns. The values of moments presented are average values over lengths one tenth of the distance between the columns.

CHAPTER III

OUTLINE OF ANALYSIS

3.1 General Remarks

Although there are many variables that influence the behavior of any structural system, only those thought to have the greatest effect were considered in the investigation. The following sections contain the numerical values of each of the parameters considered.

3.2 Overhang Ratio

Preliminary investigation showed that sufficient accuracy could be obtained by dividing the slab between the columns into ten equal grid spacings as shown in FIGURE 1. The lengths of the cantilevered or overhang portion of the slab were multiples of this same grid spacing and were expressed as dimensionless ratios of the length of overhang to the column spacing.

It is not likely that a slab with an overhang of less than 0.2 times the distance between the columns would be considered cantilevered nor that one with an overhang ratio greater than 0.5 would be constructed. Thus in this investigation overhang ratios of 0.2, 0.3, 0.4 and 0.5 were considered.

3.3 Loading Conditions

Two types of symmetrical loading arrangements were considered:

(a) A unit uniformly distributed load, q, over the entire slab including the cantilevered portion. This loading represents the most frequently specified design live load as well as representing the dead weight of the slab.

(b) A unit line load, p, around the perimeter of the slab. This loading represents the dead weight of the beam and any exterior walls.

Since the solutions were elastic and superposition is possible, the above loadings may be combined in various ratios to obtain many practical loadings.

3.4 Beam Variable

The flexural stiffness of the edge beam is represented by a dimensionless ratio, H, which is defined as the flexural stiffness of the beam divided by the flexural stiffness of one panel width of slab; $H = E_b I_b / Nb$. It is obvious that a larger value of H represents a beam with a greater flexural stiffness.

Similarily the torsional stiffness on the beams is represented by a dimensionless ratio, J, which is defined as the torsional stiffness of the beam divided by the torsional stiffness of one panel width of slab; $J = CG/Nb$.

The flexural and torsional stiffnesses of the beams were varied separately. In order to obtain a range of results, numerical values of H equal to 0, 3 and 10 and J equal to 0, 0.5 and 1 were chosen. For an overhang ratio of 0.3 all nine combinations of these values of H and J

were examined, while for the other ratios of overhand only the four combinations of H=3 and 10 with J=0 and 1 along with H=J=0 were examined.

3.5 Column Variable

The magnitude of the column bending stiffness is represented by a dimensionless ratio, K, which is defined as the sum of the column stiffnesses above and below the slab divided by the flexural stiffness of one panel width of the slab.

Square columns of zero and infinite stiffness were considered for all the examined combinations of the other variables. In order to obtain some means of interpolating for intermediate column stiffnesses, K values equal to 5, 10, 20, 30, 40 and 50 were examined for specific cases.

3.6 Poisson's Ratio

In the majority of cases considered in this investigation the value of Poisson's ratio was zero. However, for comparitive purposes, solutions were obtained using Poisson's ratio equal to 0.3 for both loadings when OH=0.3 H=J=0 and $K=\infty$ and when OH=0.3, H=10, J=1 and $K=\infty$.

CHAPTER IV

PRESENTATION AND DISCUSSION OF RESULTS

4.1 Introduction

The numerical results obtained in this investigation include: slab and beam deflections, slab bending moments, beam bending and torsional moments, and column bending moments. The sum of the moments across all grid lines of the cantilevered portion were compared with those computed from statics. Each of the above, when applicable, was calculated for every point on the slab grid, shown in FIGURE 7, for each solution. In order to illustrate the general behavior of the system, it was necessary to reproduce only a small portion of this data.

The symmetry of the slab investigated enables most of the results to be presented as effects occuring along the one half length of grid lines running parallel to the X-X axis designated in FIGURE 7. For example, the deflection and m_y at point (7,3) are the same as the deflection and m_x at point (3,7).

Initially, the deflections and bending moments for zero values of column and beam stiffnesses are considered. The modifications due to finite stiffnesses are introduced once the basic behavior has been presented.

Positive deflections are downward, and positive moments cause compression on the top of the slab.

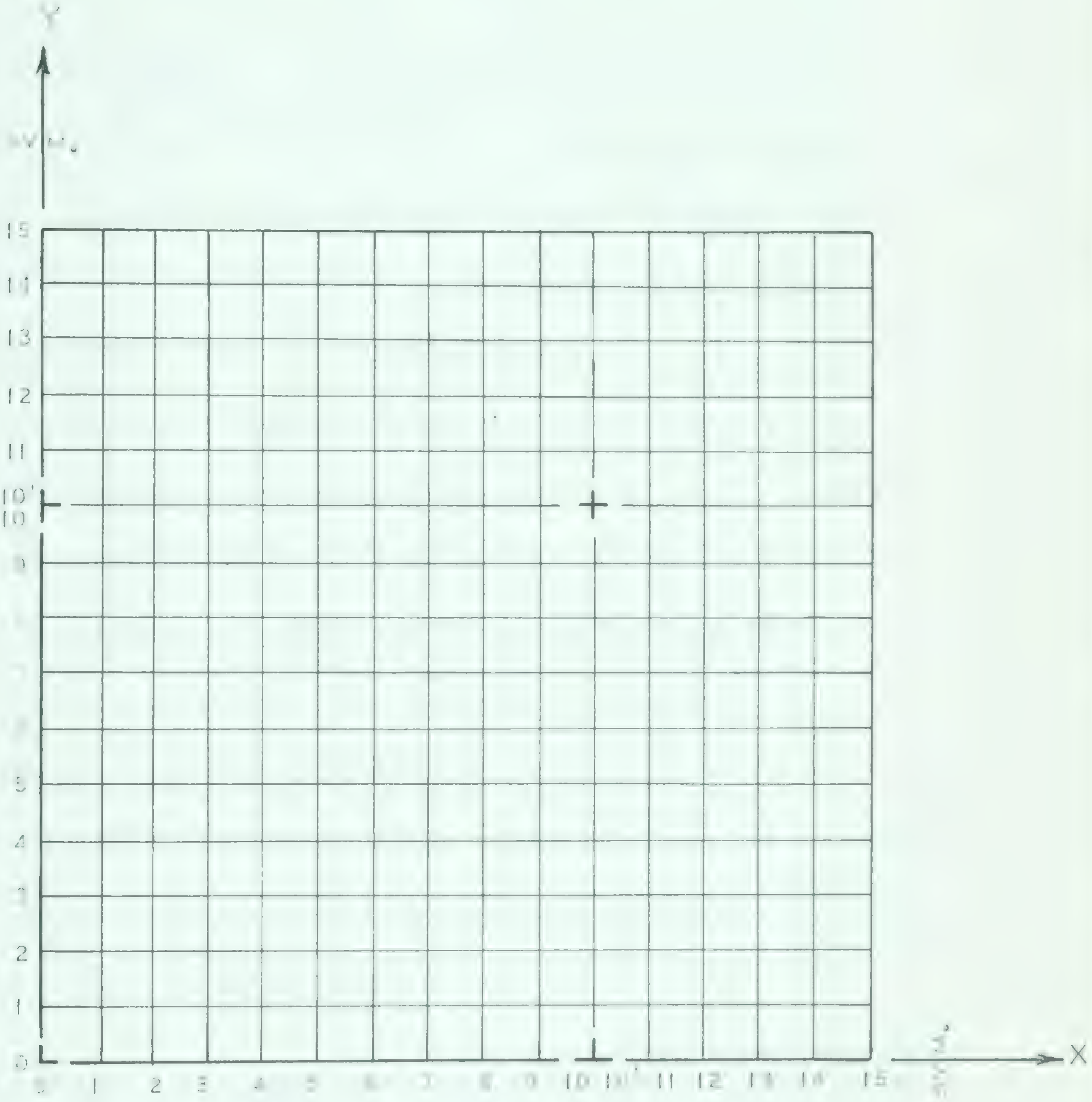


FIGURE 7 - GRID SYSTEM FOR LOCATING POINTS ON THE SLAB

4.2 Uniform Load Deflections

The deflection pattern of the cantilevered slab being investigated is fundamentally a function of the ratio of the length of overhang to the distance between the columns. Considering first the case of zero column and edge beam stiffnesses, FIGURE 8 shows the deflections along the diagonal (0,0) to (15,15). For the large overhang ratios it is apparent that the load on the cantilevered portion causes the corner of the slab to sag substantially. For example, with an overhang ratio of 0.5 the corner deflection is 8.4 times as much as the maximum interior deflection of 0.0035 qb^4/N. Reducing the overhang ratio does not affect the load on the interior panels of the slab but decreases the total load on the cantilevered portion. This results in a reduction in the cantilever deflections and an increase in those of the interior panels. For overhang ratios smaller than approximately one third, parts of the cantilevered portion actually rise. The corner, which exhibits the greatest range of deflections for changes in the overhang ratio, rises 5.5 times the maximum interior deflection of 0.0095 qb^4/N when the overhang ratio is 0.2.

FIGURE 9 shows the edge deflections for the overhang ratios considered. The deflections along the other grid lines, parallel to the edge 15-X, in the cantilevered portion are similar to those of the edge, except that they are numerically smaller and dip more between the column lines. This behavior is illustrated for an overhang ratio of 0.5 in the deflection contours of FIGURE 10. The numerical results for the other overhang ratios can be interpolated from the edge deflection in FIGURE 9 and the deflections along 11-X in FIGURE 11.

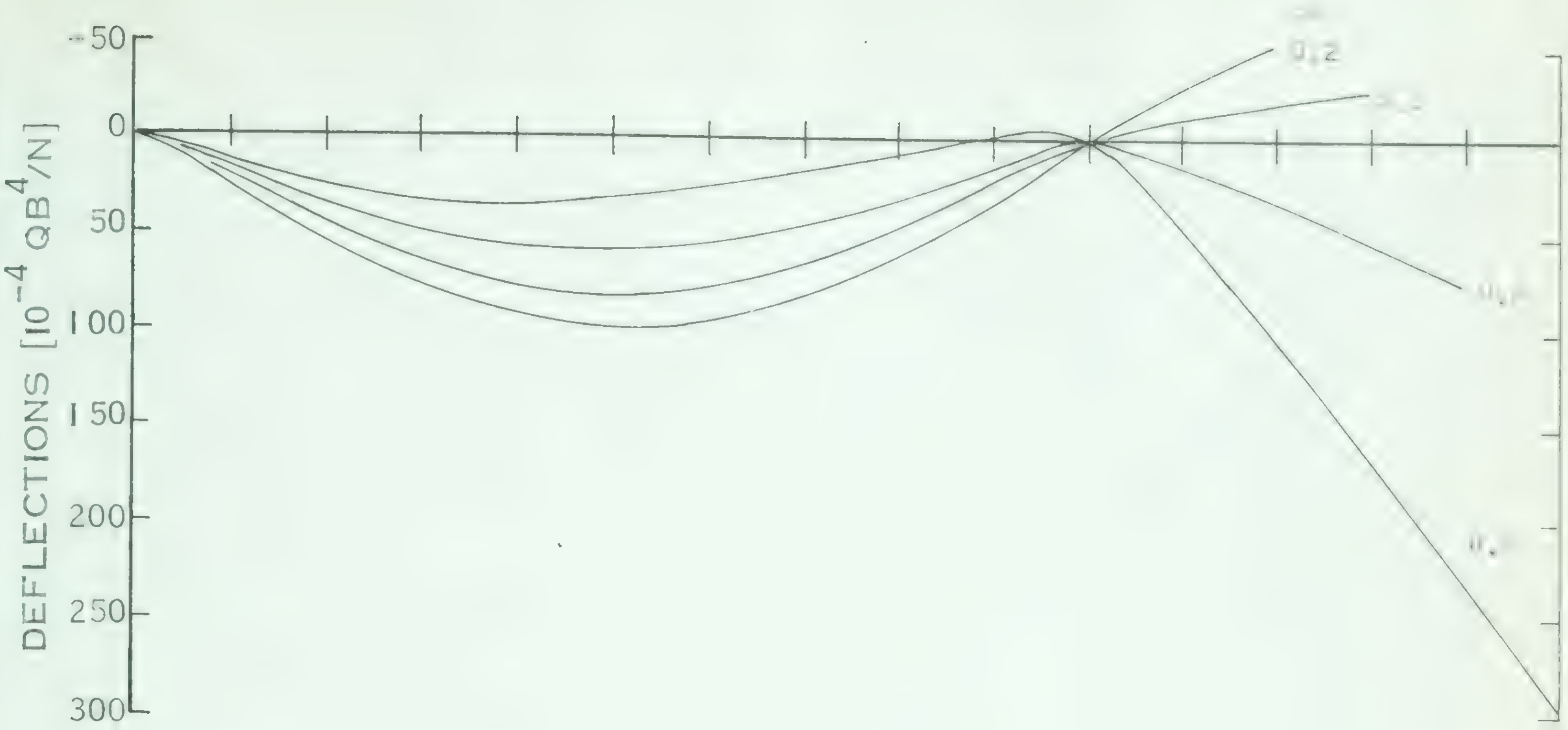


FIGURE 8 – SLAB DEFLECTIONS ALONG DIAGONAL
UNIFORM LOAD, H = 0, J = O, K = O

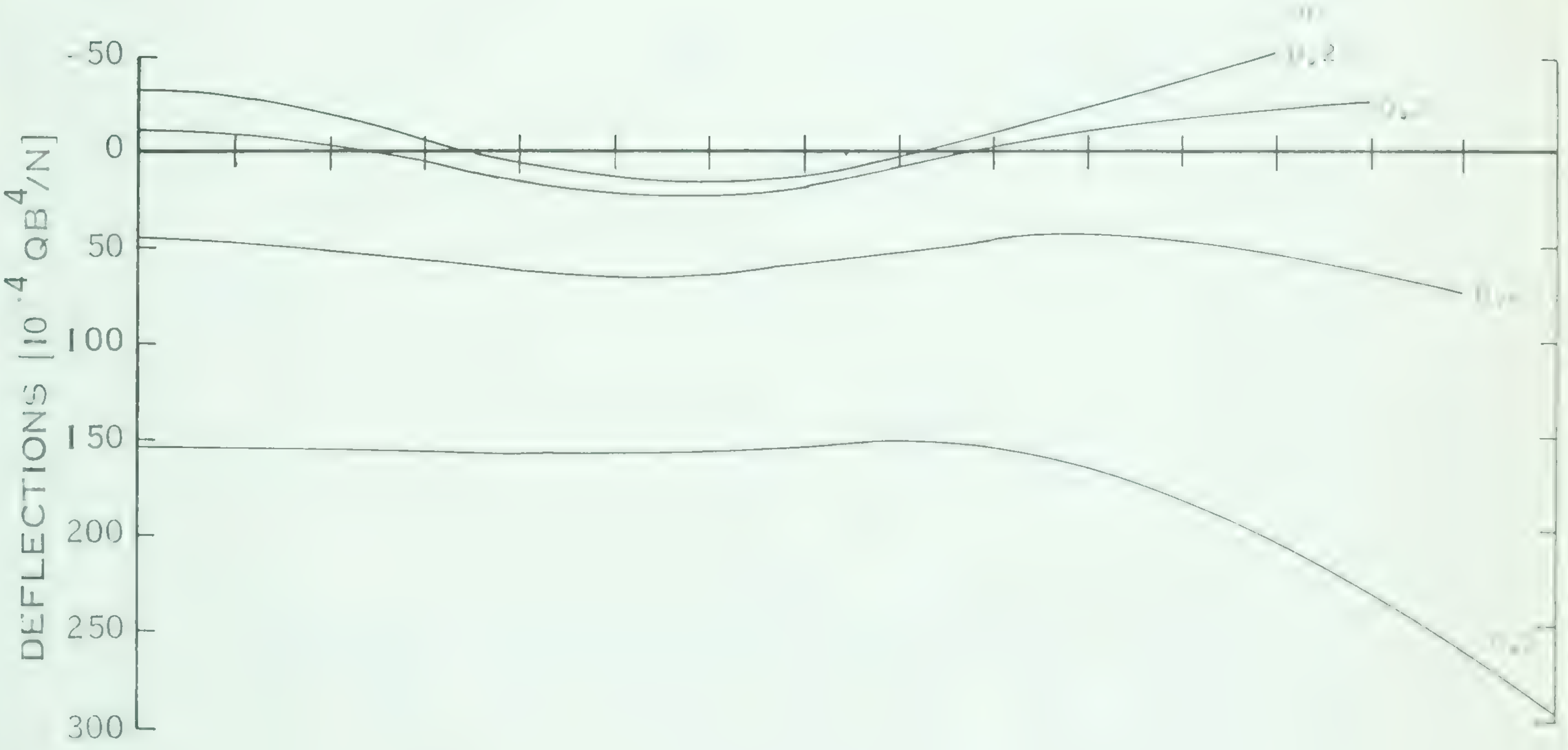


FIGURE 9 – SLAB EDGE DEFLECTIONS
UNIFORM LOAD, H = 0, J = 0, K = 0

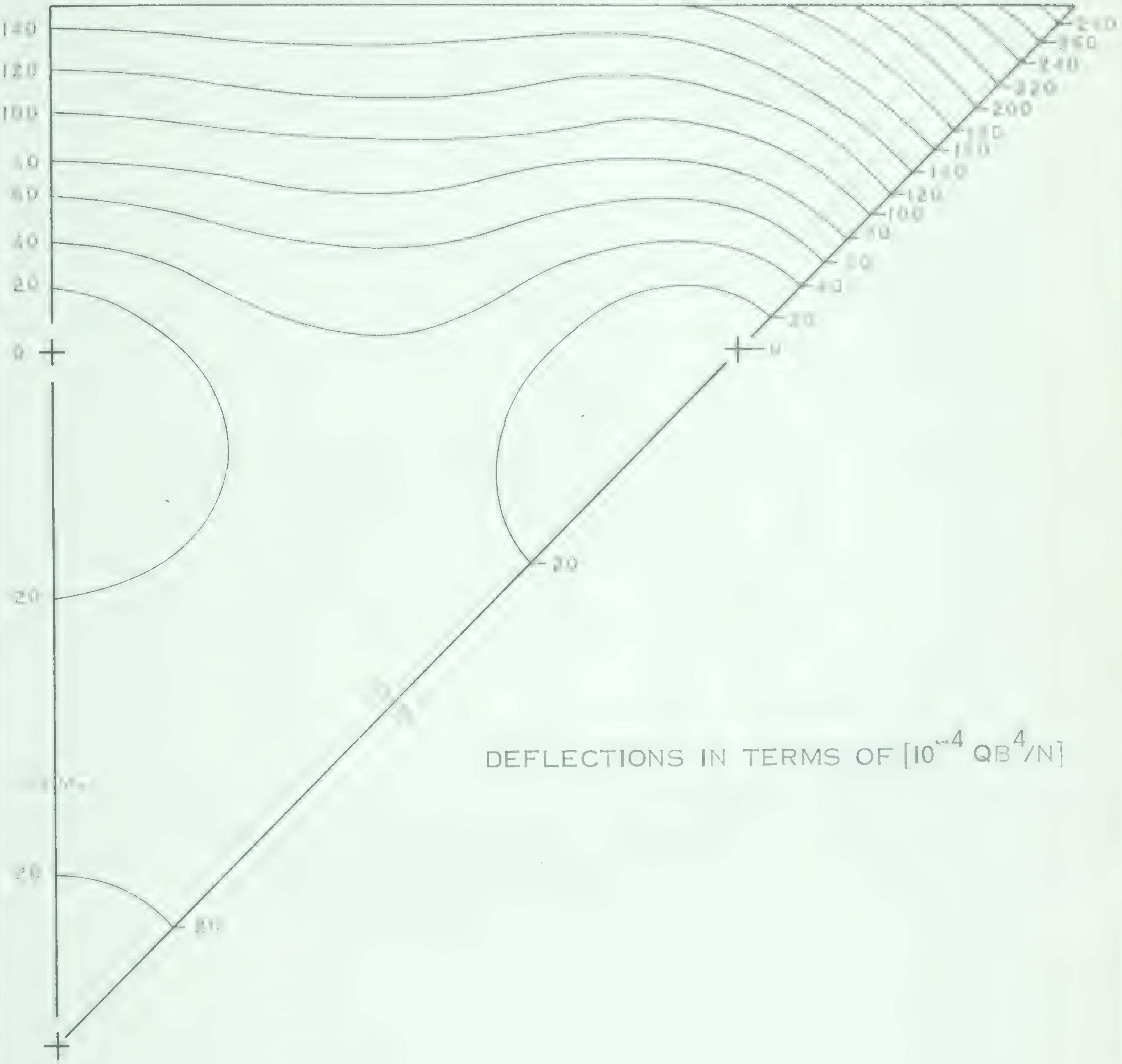


FIGURE 10 - DEFLECTION CONTOURS
UNIFORM LOAD, OH = 0.5, H = 0, J = 0, K = 0

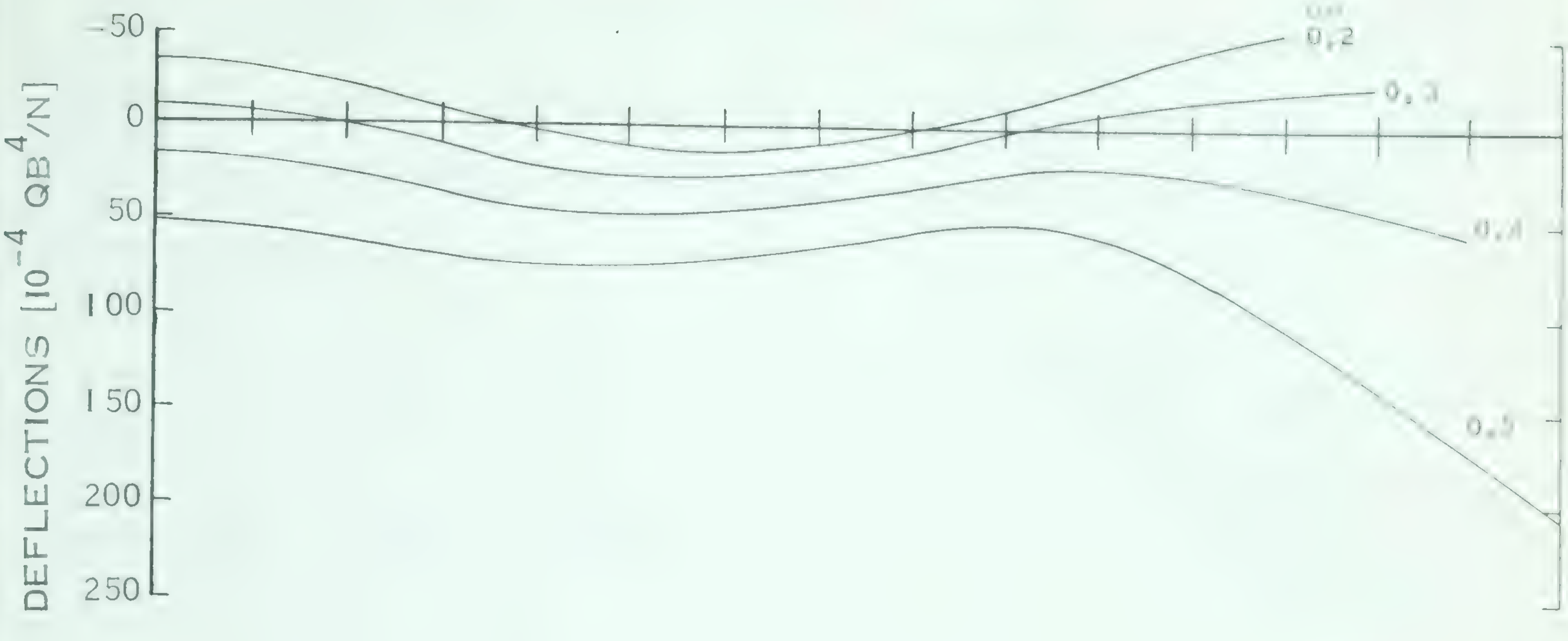


FIGURE II - SLAB DEFLECTIONS ALONG II-X
UNIFORM LOAD, H = 0, J = 0, K = 0

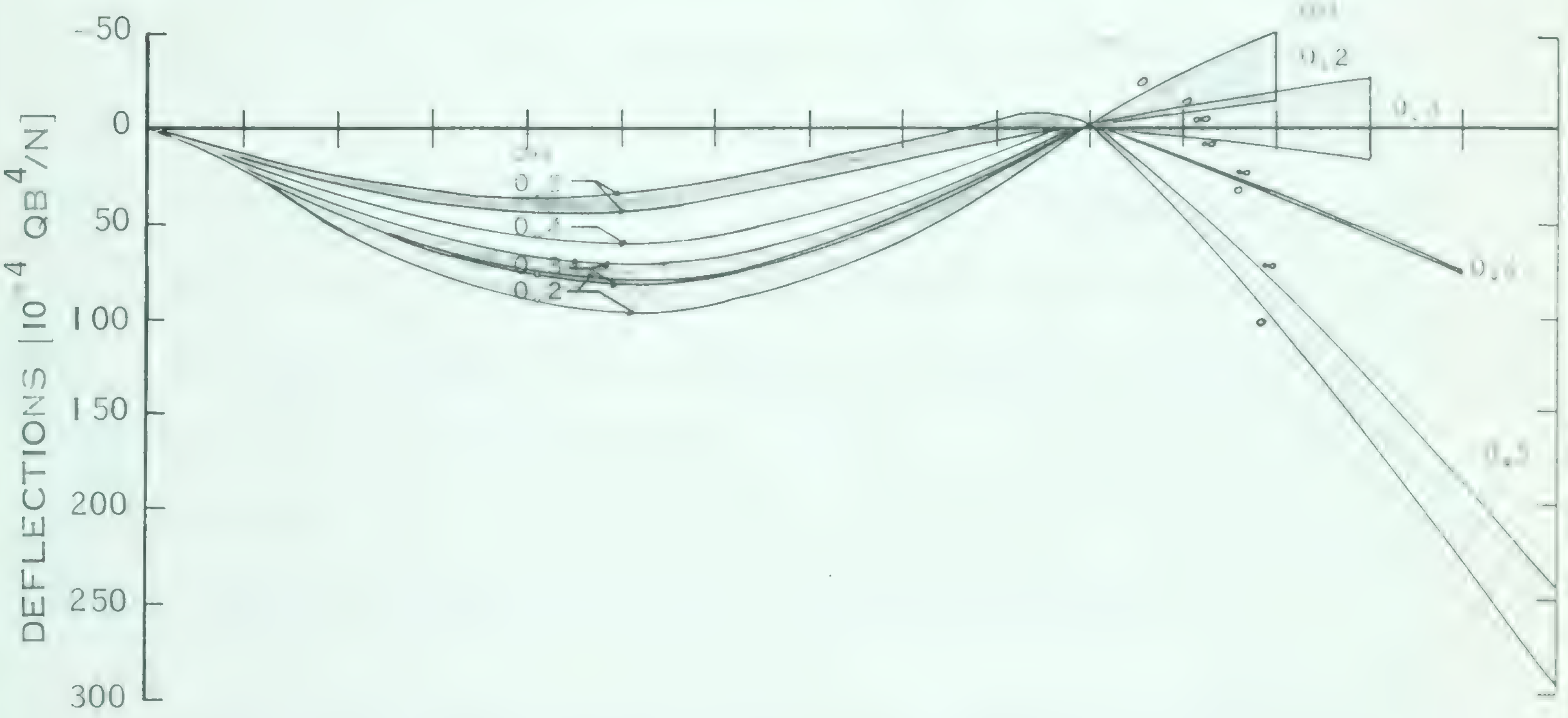


FIGURE I2 - SLAB DIAGONAL DEFLECTIONS
UNIFORM LOAD - H = 0, J = 0

A reduction in the overhang ratio reduces the cantilever and increases the interior deflections along corresponding grid lines. FIGURES 8 and 11 show this effect for the diagonal (0,0) to (15,15) and grid line 11-X respectively.

The general pattern of deflections along any of the grid lines may be predicted by considering a uniformly loaded continuous beam strip simply supported over the column lines. The magnitude of the maximum deflections, however, may be 100% in error.

When a column has finite bending stiffness, K, the tendency of the slab to slope over it is resisted by a couple exerted by the column. The magnitude of this couple, which reduces the slab slope at the column, becomes larger as the tendency for slopes to occur and/or the column stiffness increases. The magnitude of the slab slopes at the ends of the edge and corner columns, and thus the column rotations, are directly related to the overhang ratio. For an overhang ratio of approximately 0.4 no slopes develop over these columns. Increasing the overhang ratio above this value results in increasingly larger slopes towards the edge. Therefore, the column couples tend to reduce the cantilever deflections and increase those in the interior panels. The opposite is true for smaller overhang ratios. No couples are exerted by the columns across lines of symmetry because the slopes are zero in these directions.

The range of deflections possible in the entire slab, for values of column stiffness ranging from zero to infinity, can be determined, with reasonable accuracy, from the diagonal deflections presented in FIGURE 12.

FIGURE 13 shows the variation in the cantilever deflections for the intermediate values of column stiffness when H=J=0 and OH=0.3. The distribution is similar for other values of H, J, and OH. From this and the definition of K it is apparent that the deflections, obtained using columns with stiffnesses in the range generally encountered in practice, approach those obtained using columns with infinite bending stiffness.

Increasing the edge beam stiffness, H, reduces the magnitude of, and the variation in, the edge deflections. This is demonstrated in FIGURE 14 for an overhang ratio of 0.3. In this case the variation in the edge deflection is reduced from 0.0048 qb^4/N at H=0 to 0.008 qb^4/N at H=3 and to 0.0003 qb^4/N at H=10. For an overhang ratio of 0.2 these variations are 0.0068, 0.0007, and 0.0003 qb^4/N while an overhang ratio of 0.5 they are 0.0141, 0.0040 and 0.0016 qb^4/N.

Increasing J, the perimeter beam torsional stiffness, tends to reduce the magnitude of the cantilever deflections but has little effect on their distribution. The edge deflections are reduced an average of 0.0003 qb^4/N when OH=0.2 to 0.0019 qb^4/N when OH=0.5 for an increase in J from 0 to 1. The maximum interior panel deflection is changed by a 0.00074 qb^4/N decrease for the smallest overhang, to a 0.00046 qb^4/N increase for the largest overhang.

The reduction in the edge deflections resulting from increases in H, J, and K are not directly additive. For example the corner deflection of 0.0294 qb^4/N for H=J=K=0 and OH=0.5 should be reduced approximately to 0.0076 qb^4/N for H=10, J=1 and $K=\infty$ if the effects added directly. The

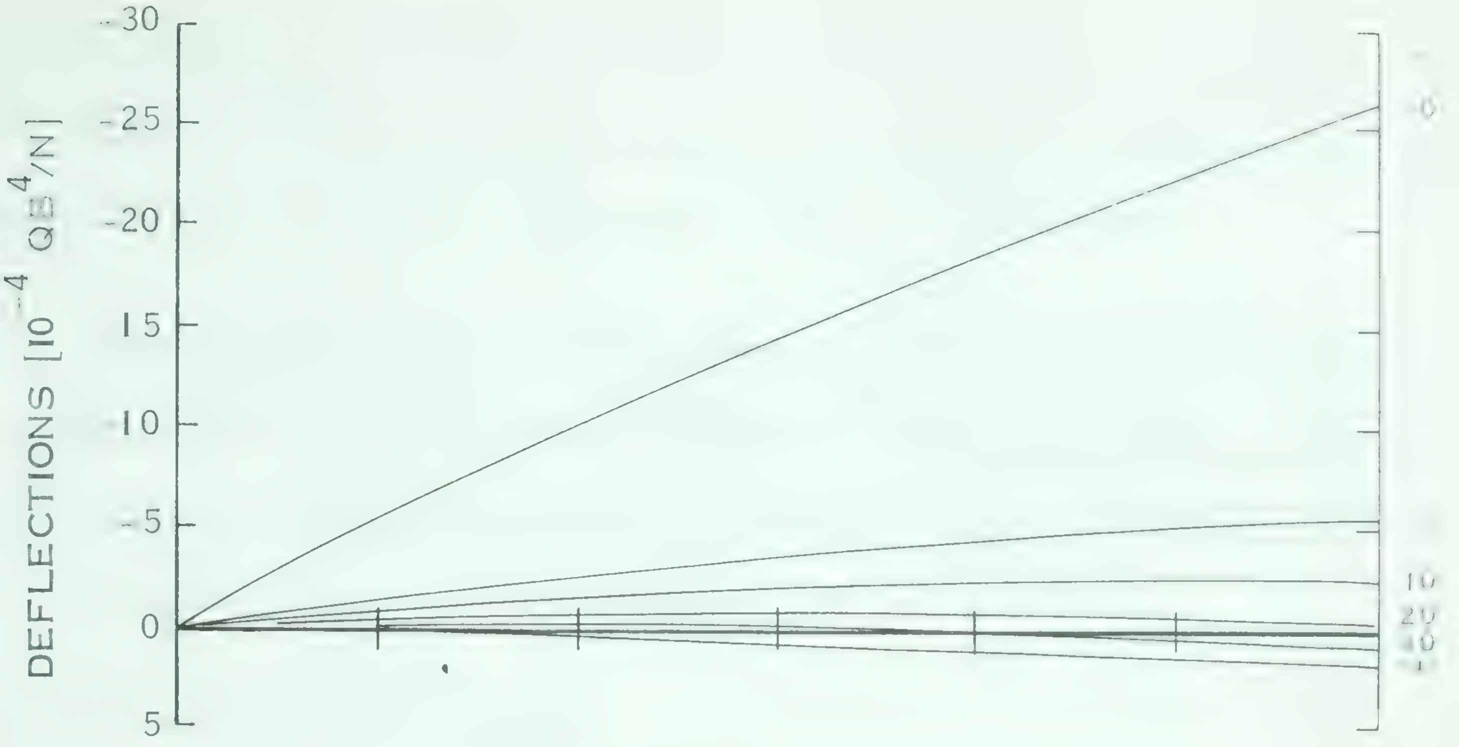


FIGURE 13 - SLAB DIAGONAL DEFLECTIONS IN CANTILEVERED PORTION UNIFORM LOAD, OH = 0.3, H = 0, J = 0

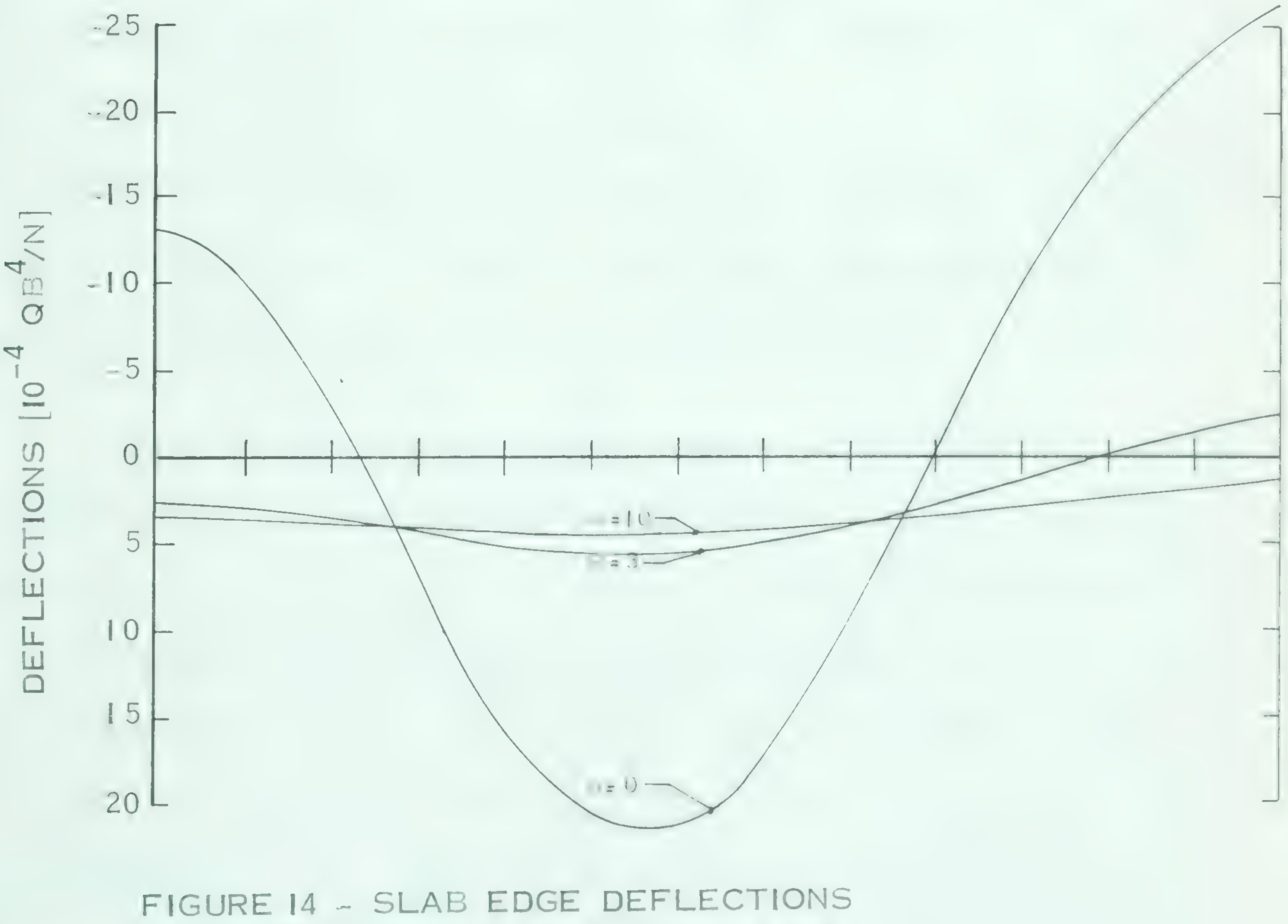


FIGURE 14 - SLAB EDGE DEFLECTIONS UNIFORM LOAD, OH = 0.3, J = 0, K = 0

actual corner deflection is 0.0112 qb^4/N for this case. Similarily for OH=0.2 the corner deflection of -0.0052 qb^4/N would be expected to be reduced to +0.0028 qb^4/N but the actual value is -0.0003 qb^4/N.

The effect of a change in the value of Poisson's ratio on the deflection pattern for the cases considered, OH=0.3, was almost negligible. For example, the maximum deflection was reduced only 0.6% when Poisson's ratio increased from 0 to 0.3 when OH=0.3, H=J=0 and $K=\infty$.

4.3 Uniform Load Bending Moments

When the values of column and beam stiffnesses are zero, the bending moment diagrams, along the grid lines parallel to the X-X axis, have the same shape as those of analogous continuous beams simply supported over the column lines. The numerical value of these moments, however, may vary substantially for different grid lines. FIGURES 16 to 19 illustrate this by presenting the values of m_x along 0-X, 5-X, 10-X, the edge and the analogous continuous beams. It is interesting to note the similarity in the moment values along 0-X and 10-X, and along 5-X and the edge. This is especially apparent for the larger overhang ratios.

The variation in the values of m_y across the grid lines in the cantilevered portion is presented in FIGURE 20 for OH=0.5. This distribution is typical of all the overhang ratios. FIGURE 20 shows that the moment across 14-X humps towards positive values near points (0,14) and (10,14). This requires that larger negative moments exist elsewhere in order that the sum of the moments across the grid lines agree with statics. The cause of this variation can be explained as follows: FIGURE 9 shows that for an overhang ratio of 0.5 the deflection along the edge between points (0,15)

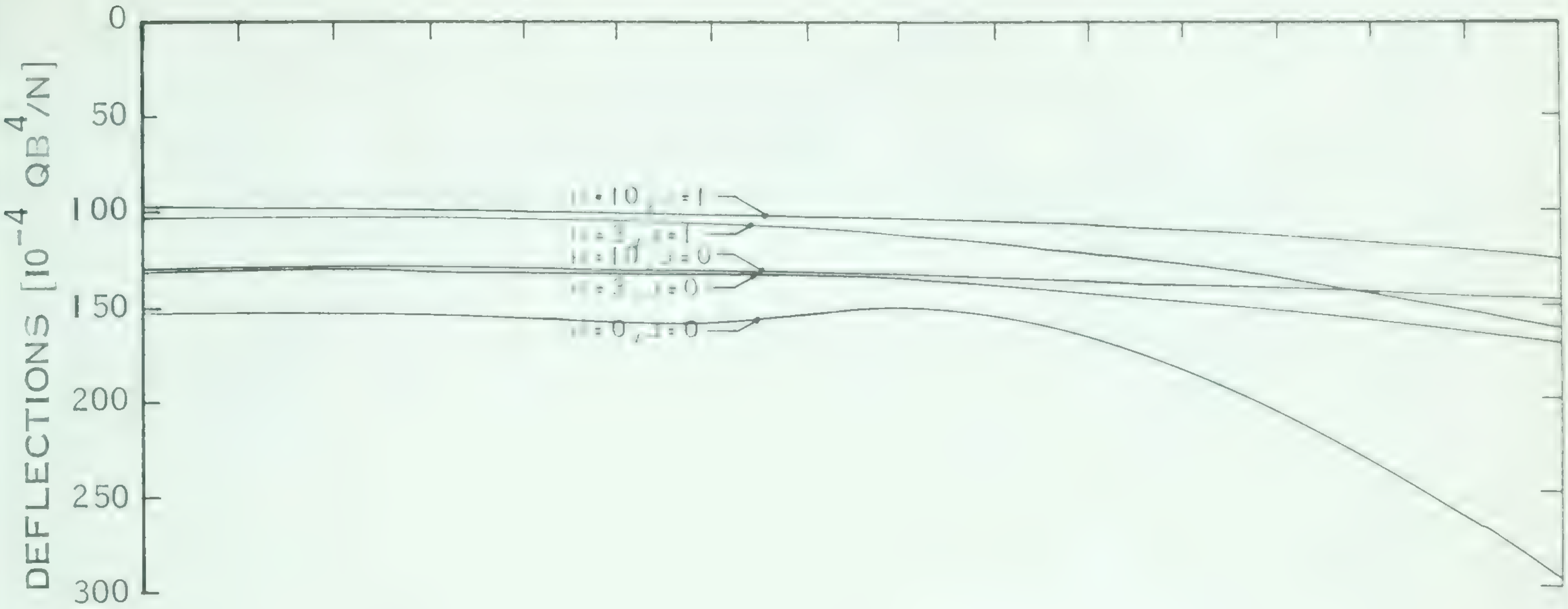


FIGURE 15 - SLAB EDGE DEFLECTIONS
UNIFORM LOAD, OH = 0.5, K = 0

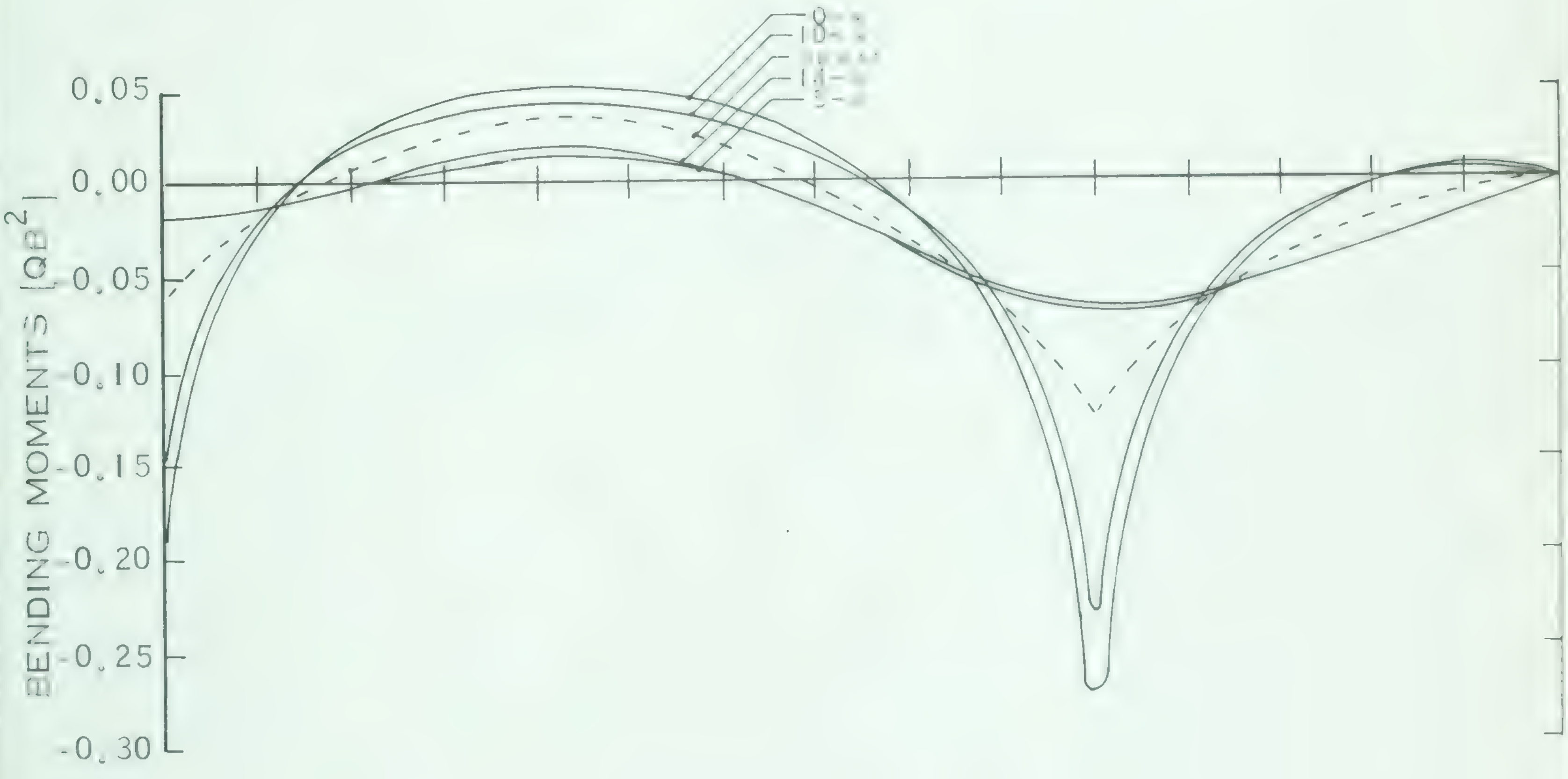


FIGURE 16 - SLAB BENDING MOMENTS
UNIFORM LOAD, OH = 0.5, H = 0, J = 0, K = 0

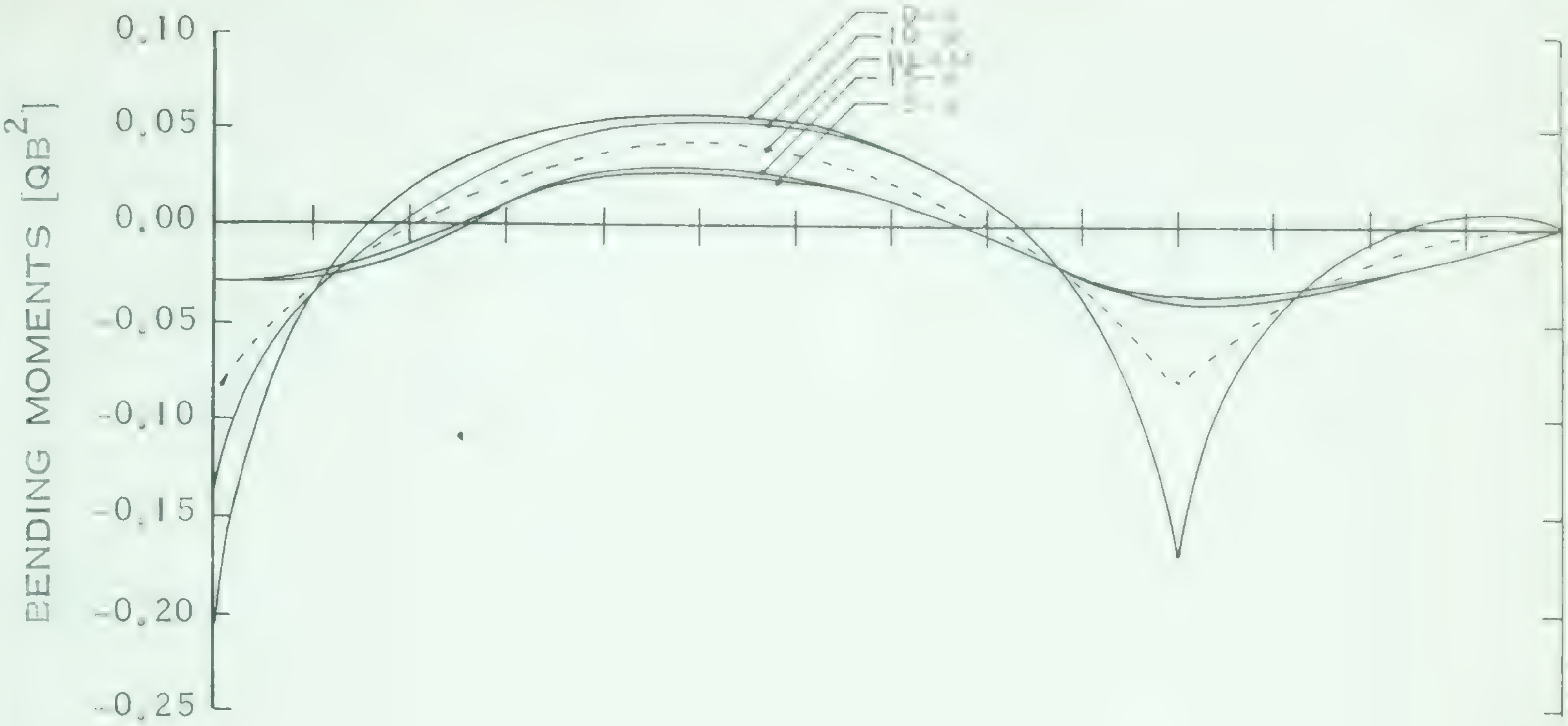


FIGURE 17 - SLAB BENDING MOMENTS
UNIFORM LOAD, OH = 0.4, H = 0, J = 0, K = 0

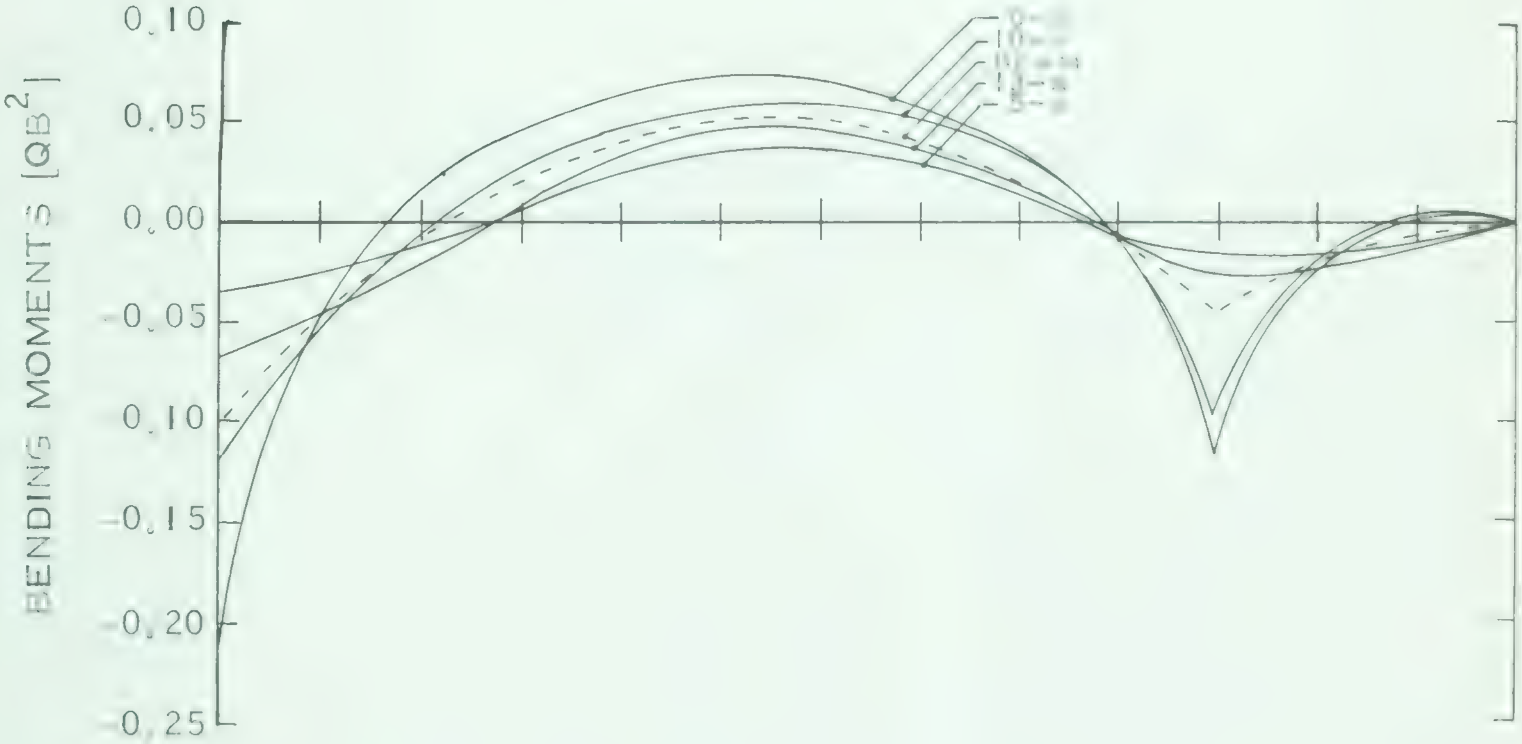


FIGURE 18 - SLAB BENDING MOMENTS
UNIFORM LOAD, OH = 0.3, H = 0, J = 0, K = 0

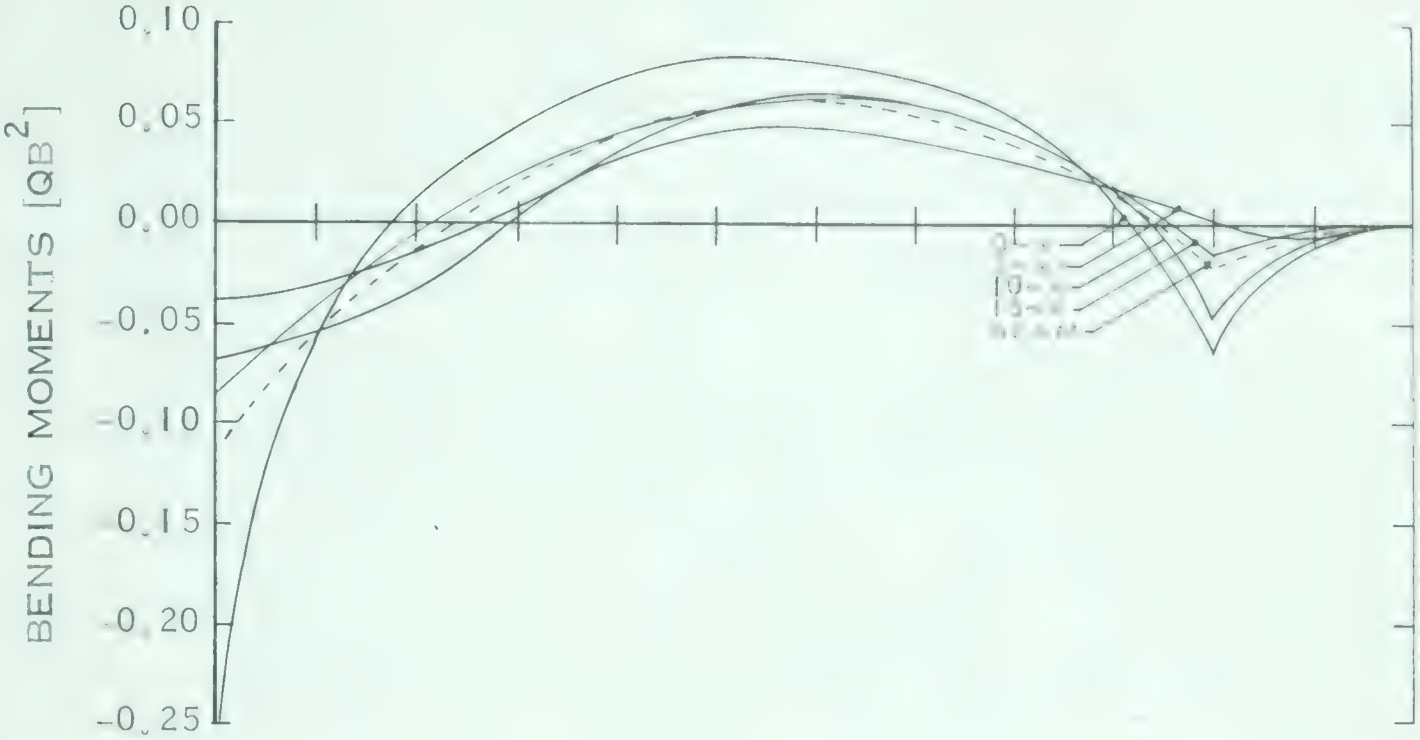


FIGURE 19 - SLAB BENDING MOMENTS
UNIFORM LOAD. OH = 0.2, H = 0, J = 0, K = 0

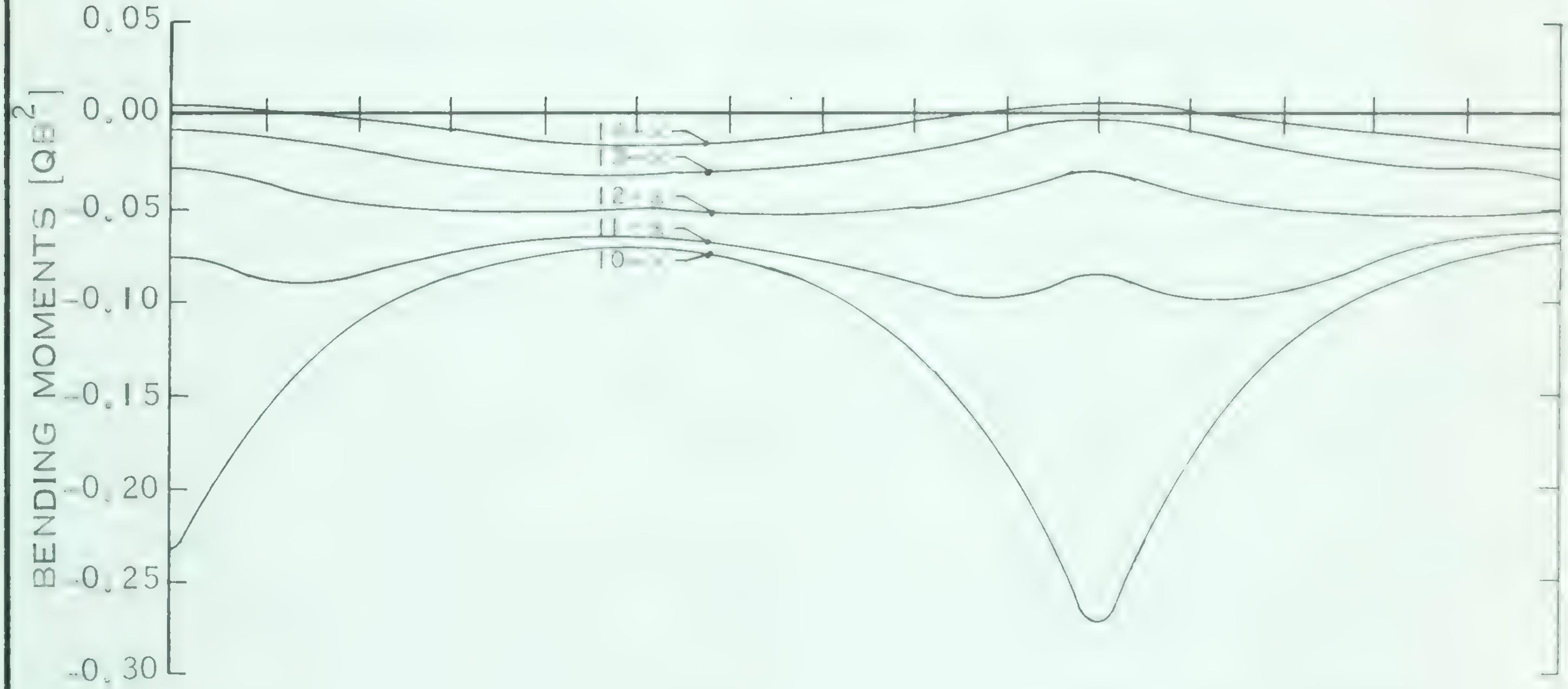


FIGURE 20 - SLAB MOMENTS ACROSS CANTILEVERED SECTION
UNIFORM LOAD, OH = 0.5, H = 0, J = 0, K = 0

to (10,15) is nearly constant and FIGURE 10 shows that the slab sags between the columns along the grid line 10-X. This requires that the average slope between (0,10) to (0,15) and (10,10) to (10,15) be greater than between (5,10) and (5,15). Considering the slab between 14-X and 15-X as a beam with torsional stiffness equal to the slab it replaces, there will be twisting moments developed which tend to equalize the slopes across grid line 14-X. This action causes positive moments to be induced near points (0,14) and (10,14) and negative moments near (6,14). Across grid line 14-X, for OH=0.5, the moments range from 144% positive to 256% negative of the average static moment which is -0.005 qb^2.

With smaller overhangs the fact that the slab sags more between the columns than along the edge, results in similar distributions of moments. However, the extent of the variations is not as great as with the large overhangs. For example, across grid line 12-X, which is one grid unit in from the edge for OH=0.3, the moments vary from 82% positive to 164% negative of the average static moment.

The sharp curvatures in the slab over the colum lines produce a stiffening effect which attracts moment. This is especially true in the immediate vicinity of the columns as seen in FIGURE 20. The maximum negative across 10-X is increased to 143% of the respective average static moment for OH=0.2 to 217% for OH=0.5 while the minimum negative moment is reduced to -10% and 57% respectively. Across the intermediate grid lines in the cantilever portion the moments vary between the extremes at the column line and the edge as shown in FIGURE 20.

The variation in moment across the grid lines in the interior

of the slab follows patterns similar to that along 10-X. The extent of these variations can be seen in FIGURES 16 to 19. Grid lines 9-X, 8-X and 1-X exhibit humps toward positive moment. These are more localized and smaller in magnitude than those in the cantilever section, even though they result from the same twisting actions.

The effect of increasing, the column bending stiffness, K, on the slab bending moments is most pronounced when there is a tendency for slopes to develop over the columns. The direction of the concentrated couple applied by the column is such that the slopes are reduced. This means that for overhang ratios larger than 0.4, an increase K results in an increase in the negative moments on the exterior face, and a decrease in the negative moments at the interior face of the edge and corner columns. The opposite occurs for overhang ratios smaller than 0.4. In fact the bending moments at the exterior face of the edge and corner columns actually become positive for an overhang ratio less than approximately 0.25 when the edge beam stiffnesses are zero. FIGURE 21 shows the magnitude of the M_C moments induced in the edge and corner columns of infinite stiffness for various overhang ratios and beam stiffnesses. Increased values of beam bending and torsional stiffness reduce the column moment somewhat. The distribution for intermediate values of column stiffness is similar to that given in FIGURE 13.

The largest slab moments occur immediately adjacent to the edge columns. These, however, are localized to such an extent that the bending moments across 10-X rapidly diminish to the average static moment on both sides of the columns, regardless of the column stiffness. This is

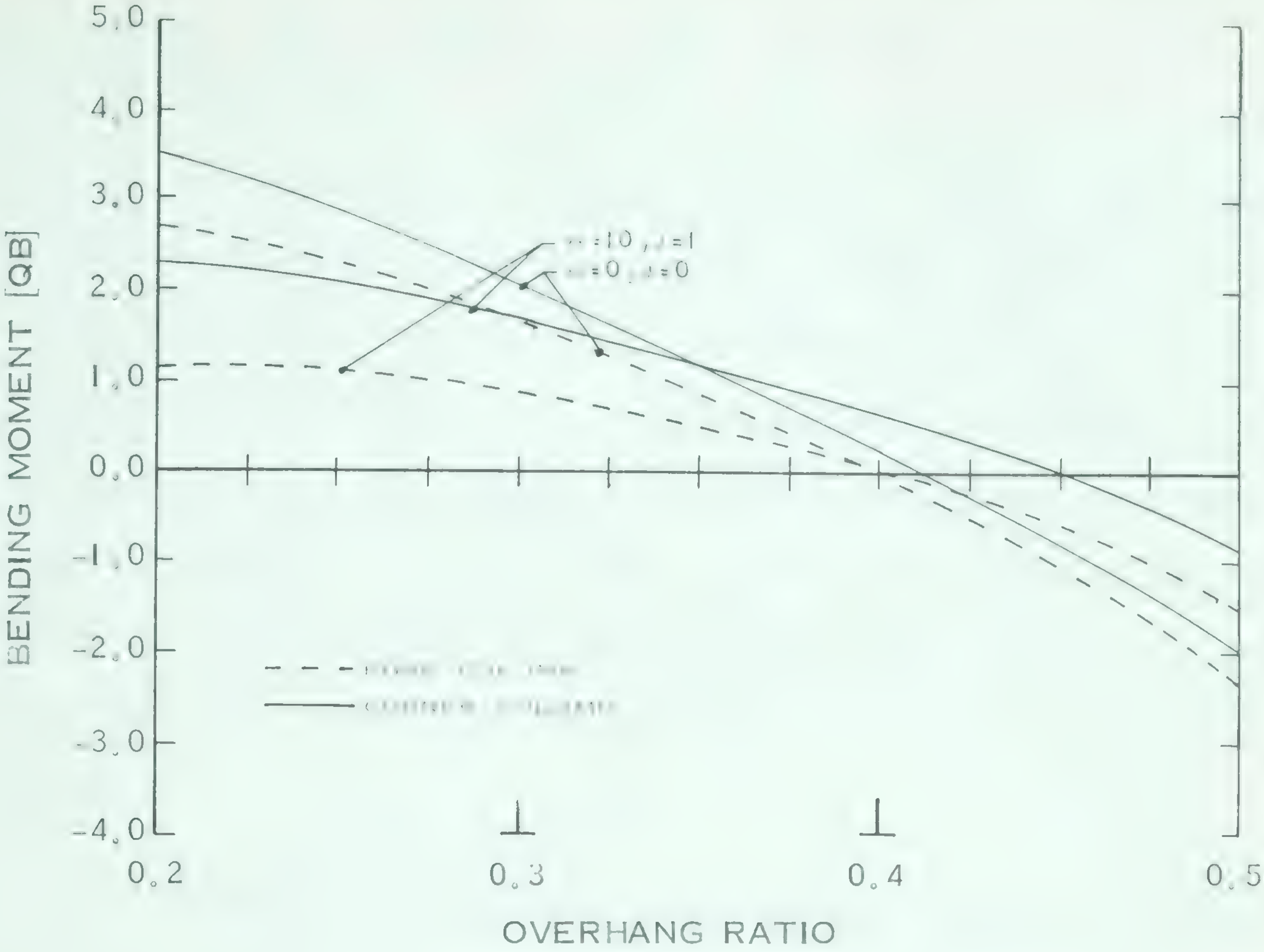


FIGURE 21 - EDGE AND CORNER COLUMN BENDING MOMENTS
UNIFORM LOAD, K = ∞

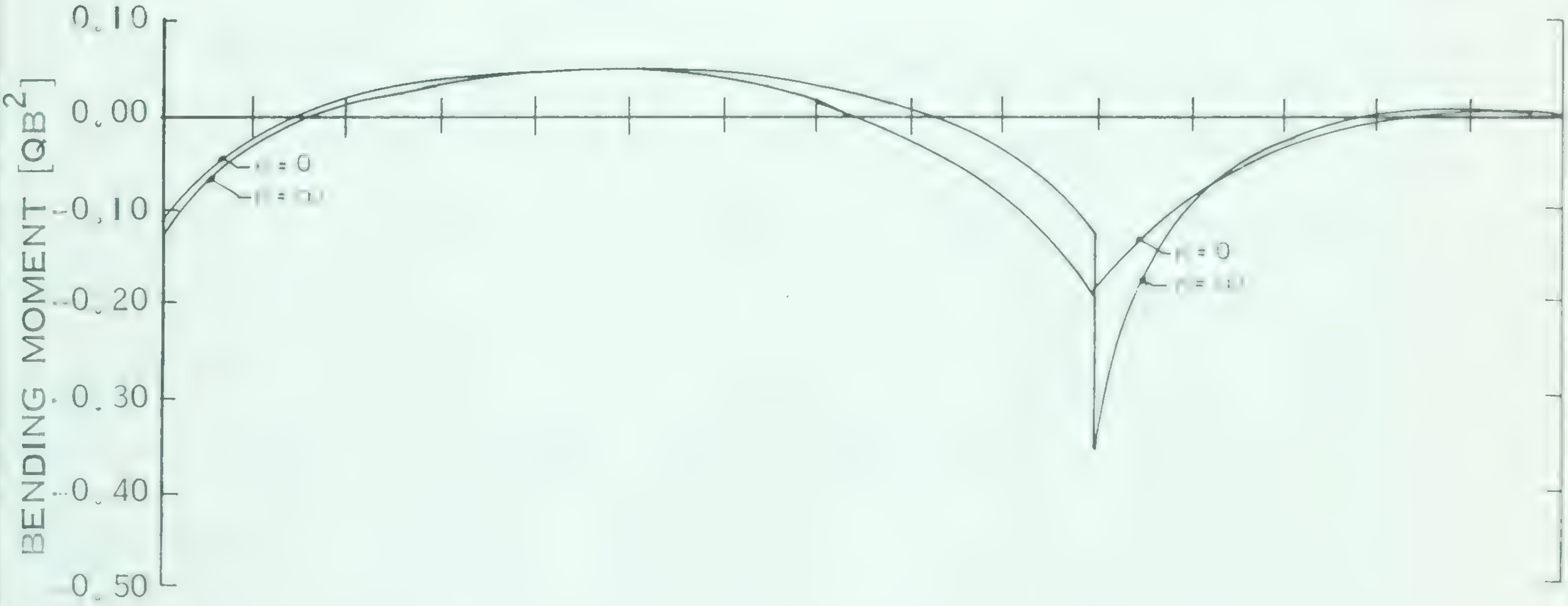


FIGURE 22 - SLAB BENDING MOMENTS ALONG 10-X
UNIFORM LOAD. OH = 0.5, H = 0, J = 0

shown for zero column stiffness in FIGURE 20 and for infinite column stiffness in FIGURE 21. The values of the moments in the slab over the column, for zero stiffnesses are shown in FIGURES 16 to 19. For infinitely stiff columns and beams of zero stiffness the maximum moments adjacent to the column at (0,10) are -0.304, -0.255, -0.187 and -0.297 and at (10,10) are -0.219, -0.206, -0.176 and -0.355 qb^2 for OH values of 0.2, 0.3, 0.4 and 0.5 respectively. The moments on the other sides of the columns can be obtained using the above slab moments and the column moments contained in FIGURE 21. For example, the bending moments at the interior forces of the edge and corner columns for OH=0.2 become plus 0.045 and 0.050 qb^2 respectively.

The presence of edge beams with bending stiffness, H, reduces the slab bending moments across any of the grid lines because the beam carries part of the total moment. The magnitudes of these beam moments are given in FIGURES 24 to 27 for various values of the overhang ratio and beam stiffness when K=0.

The portion of the total static moment not carried by the beams is distributed across the grid lines in the same manner as when H=0 except for the same modification in the region of the edge beam. The bending moment across any of the grid lines at the point of grid line intersects an edge beam is reduced to the value of the beam bending moment divided by H. This means, for example, that the distribution of the total slab moment across grid lines for OH=0.5 and J=K=0 would similar in shape to those shown in FIGURE 20 except that the moment in the region of the edge beam would be reduced as H increases.

Increasing the edge beam torsional stiffness, J, accentuates the variation in the magnitude of the positive and negative bending moments

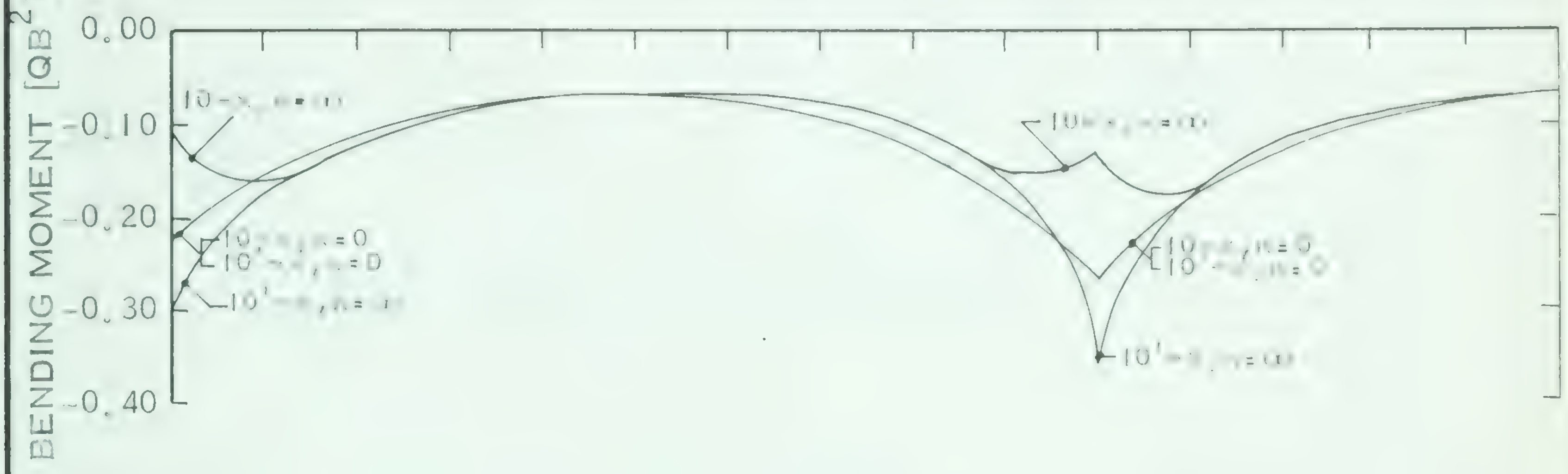


FIGURE 23 - SLAB BENDING MOMENTS ACROSS 10-X AND 10'-X
UNIFORM LOAD, OH = 0.5, H = 0, J = 0

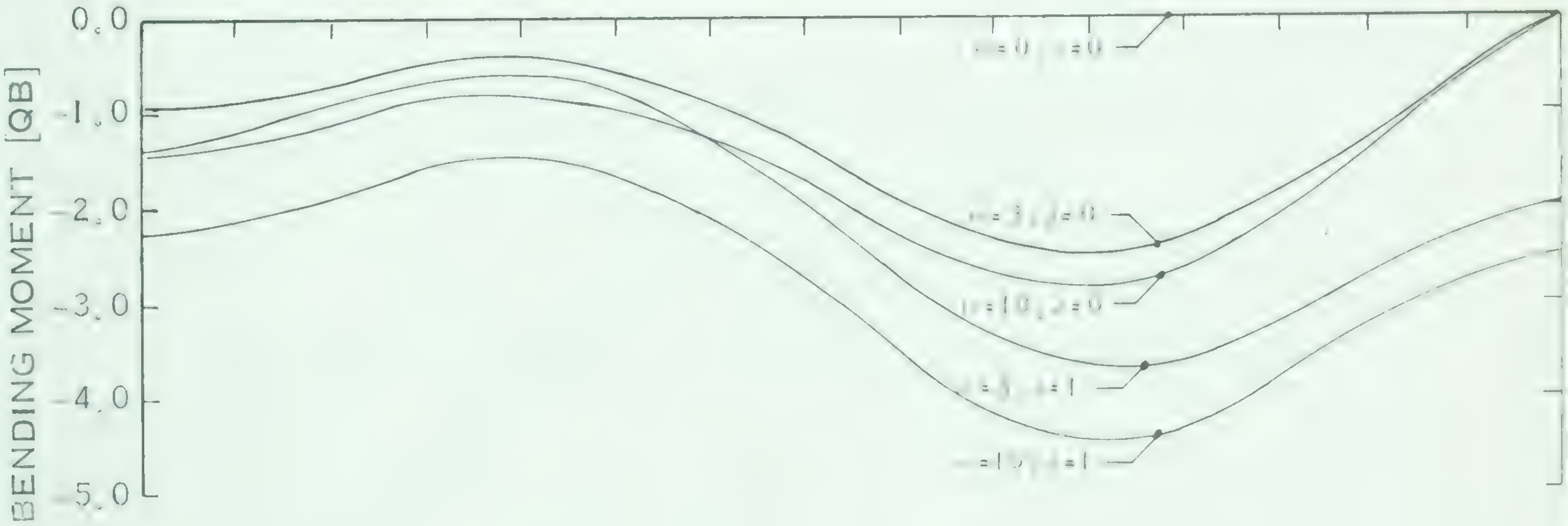


FIGURE 24 - BEAM BENDING MOMENTS
UNIFORM LOAD, OH = 0.5, K = 0

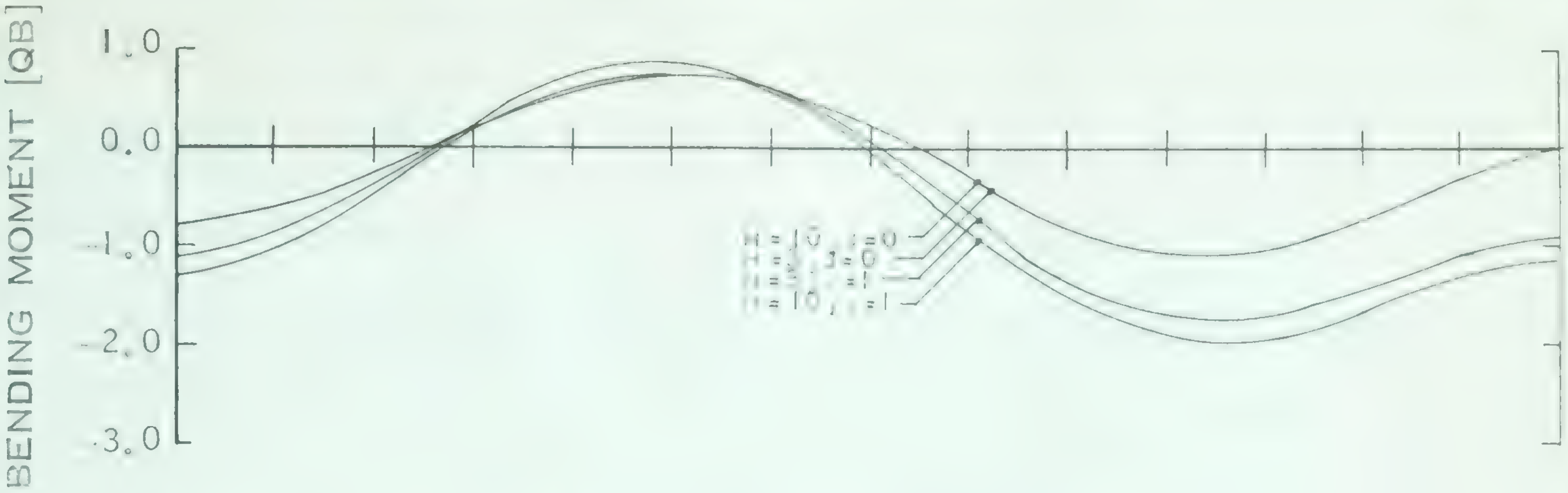


FIGURE 25 - BEAM BENDING MOMENTS
UNIFORM LOAD, OH = 0.4, K = 0

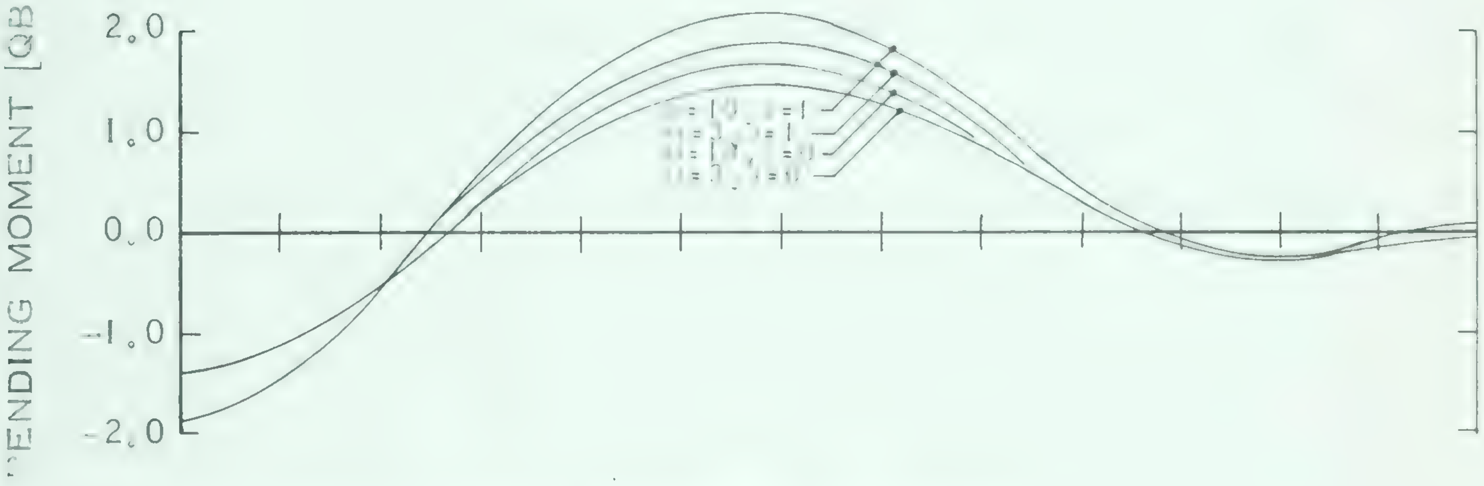


FIGURE 26 - BEAM BENDING MOMENTS
UNIFORM LOAD, 0H = 0.3, K = 0

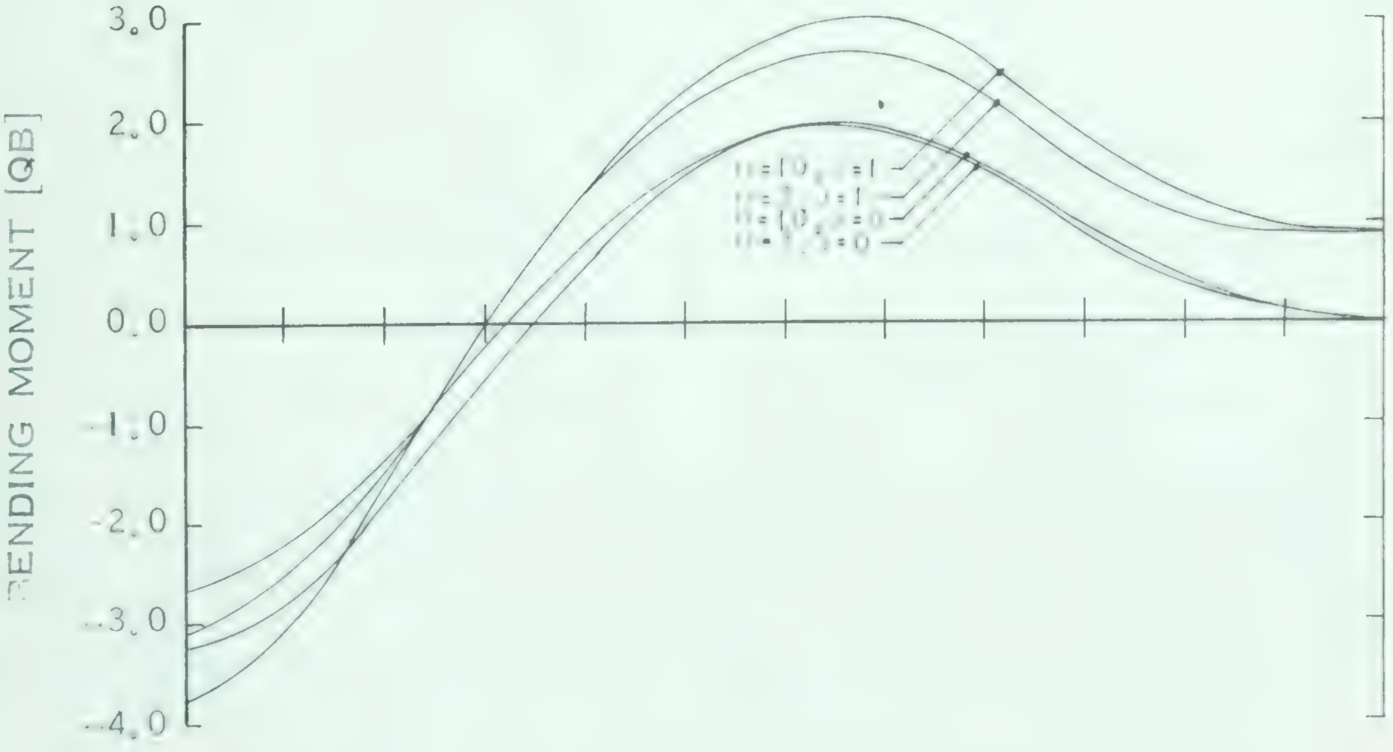


FIGURE 27 - BEAM BENDING MOMENTS
UNIFORM LOAD, OH = 0.2, K = 0

previously described as resulting from the torsional stiffness of the cantilevered portion of the slab. This increase in the variation of the bending moments across grid lines near the edge is illustrated in FIGURE 28 for grid line 13-X when OH=0.3, K=H=0 and J=0, 0.5 and 1. Similar variations result when J is varied for the other overhang ratios. It is particularily interesting to note that the value of the maximum positive moments resulting when J is large, are of the same order of magnitude as the maximum positive moments in the interior panels. For example, the maximum positive edge and interior moments that occur when J=10, H=30, K=0 and OH=0.2, 0.3, 0.4, and 0.5 are 0.0563 and 0.0702, 0.0527 and 0.670, 0.0445 and 0.0598, and 0.0578 and 0.0482 qb^2respectively. For all values of OH, H, J, and K considered, the positive moments extend furthest from the edge of the slab along the grid lines through the columns. Although the positive moments may be larger along the grid line through the edge column, they extend further from the slab edge along the grid lines through the corner columns. When J=10 the positive moments region extends approximately one third the distance from the edge to the columns for the largest overhang ratios, and all the way to the columns for the smallest overhang ratios. The effect of the different values of J can be interpreted from FIGURE 29. The magnitude of the maximum twisting moment in the edge beams is equal to the bending moment in the end of edge beams. Typical distributions of the twisting moments along the lengths of the edge beams are shown in FIGURE 30 for J, H, and K values equal to 1, 3, and 0 respectively. Decreasing H and/or J and/or increasing K reduces the maximum beam twisting moments.

Combining high values of H and J basically superimposes the effect of each with the following modification. For overhang ratios larger than approximately 0.3 the negative bending moment at the end of

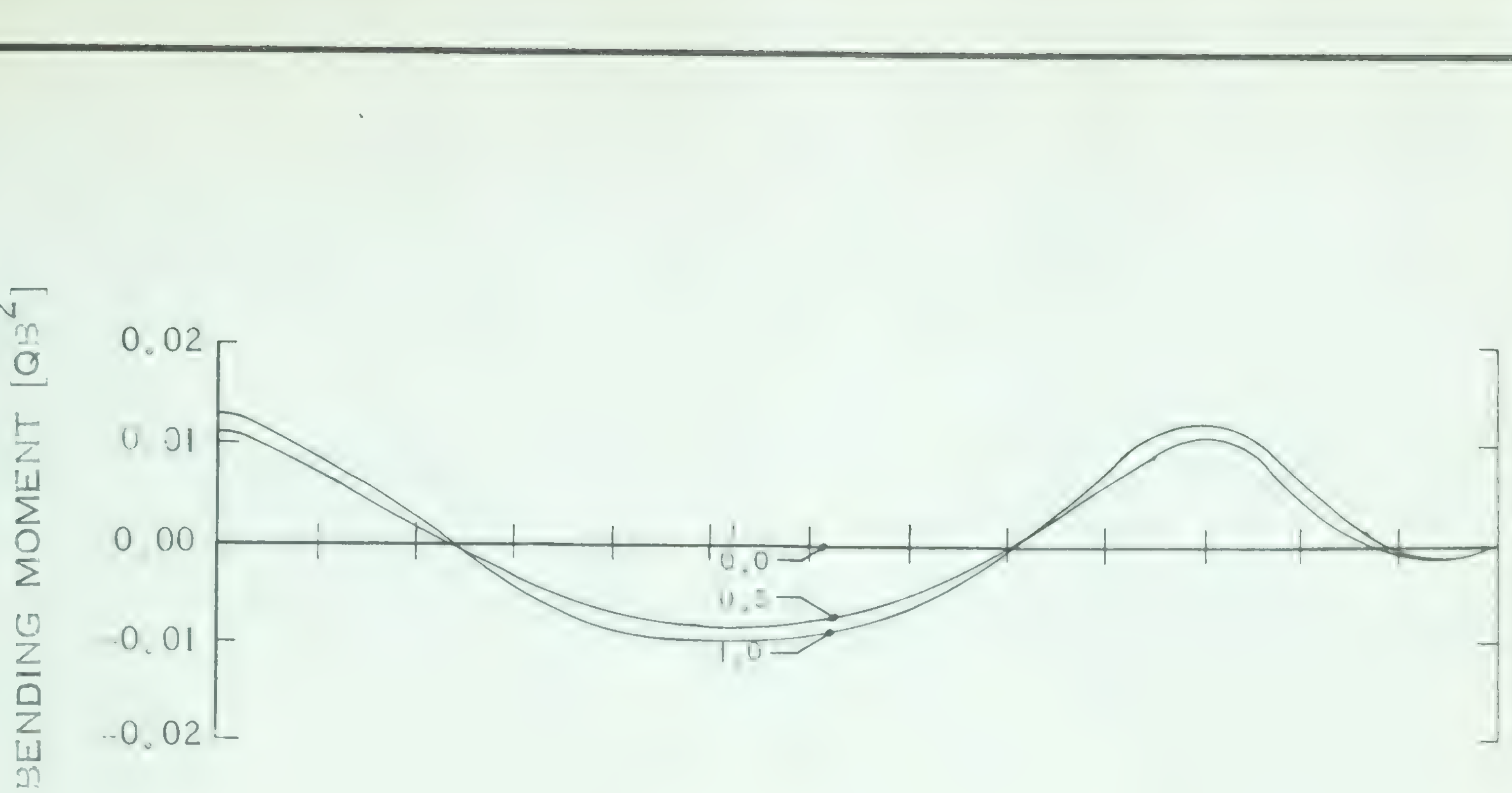


FIGURE 28 - SLAB BENDING MOMENTS ACROSS 13-X
UNIFORM LOAD, H = 0, K = 0

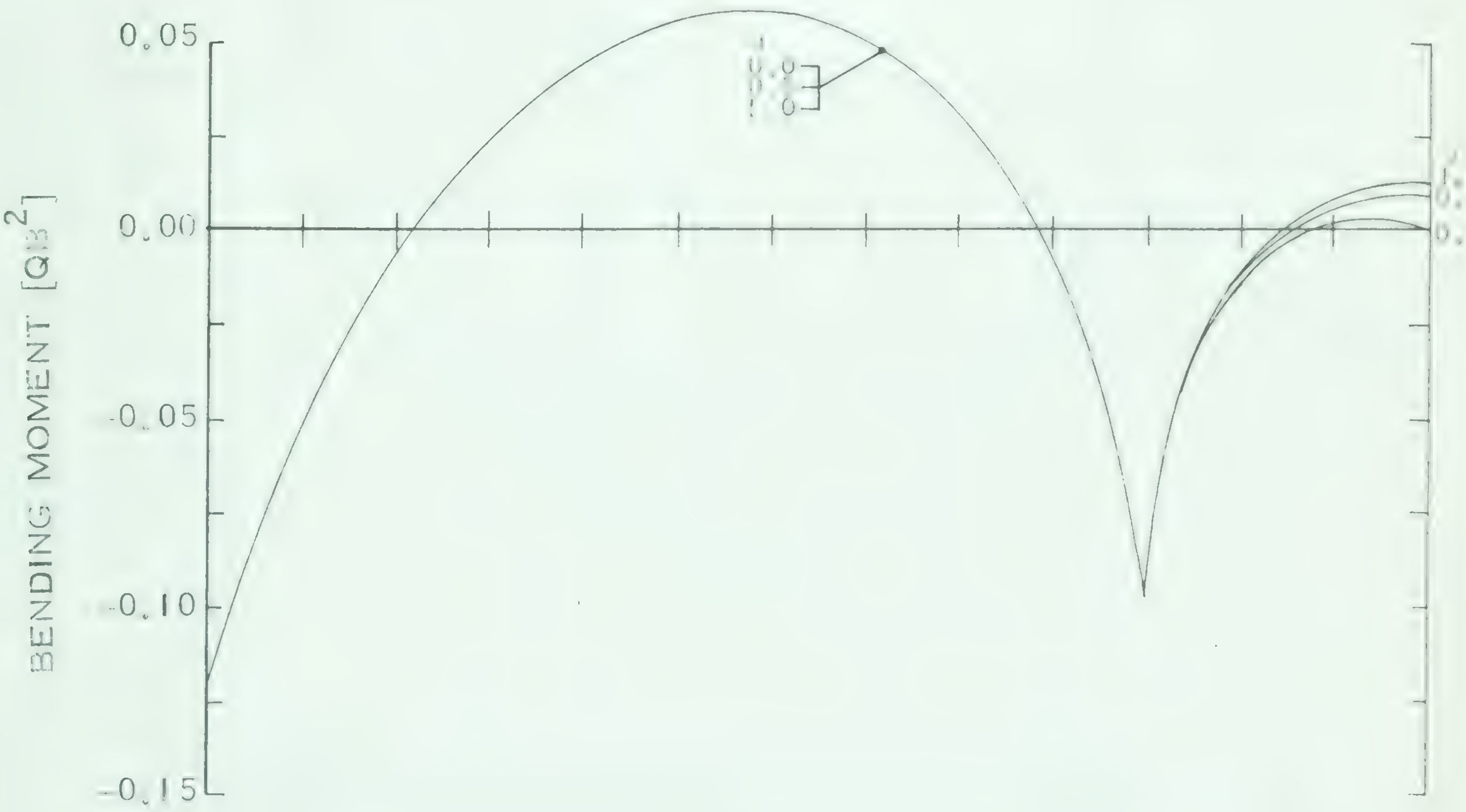


FIGURE 29 - SLAB BENDING MOMENT ALONG 10-X
UNIFORM LOAD, H = 0, K = 0

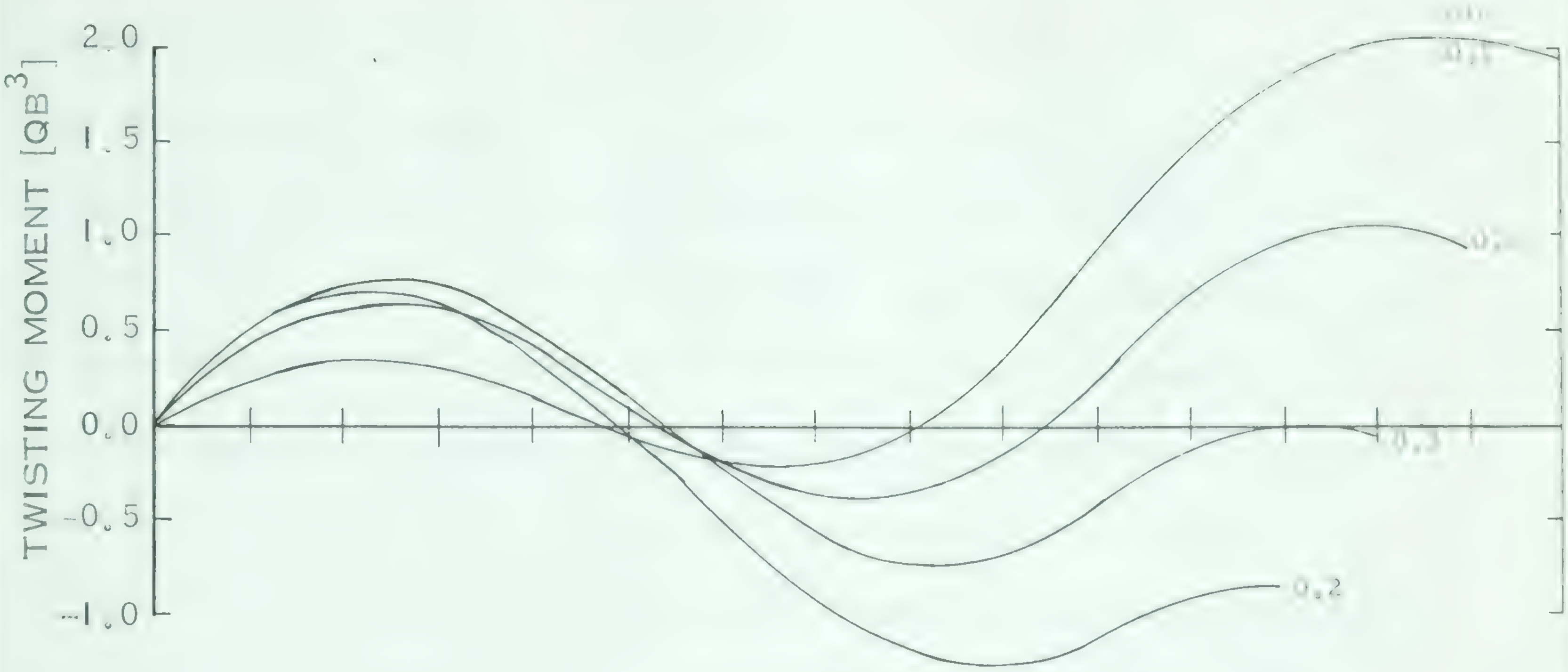


FIGURE 30 – BEAM TWISTING MOMENTS
UNIFORM LOAD, H = 3, J = 1, K = 0

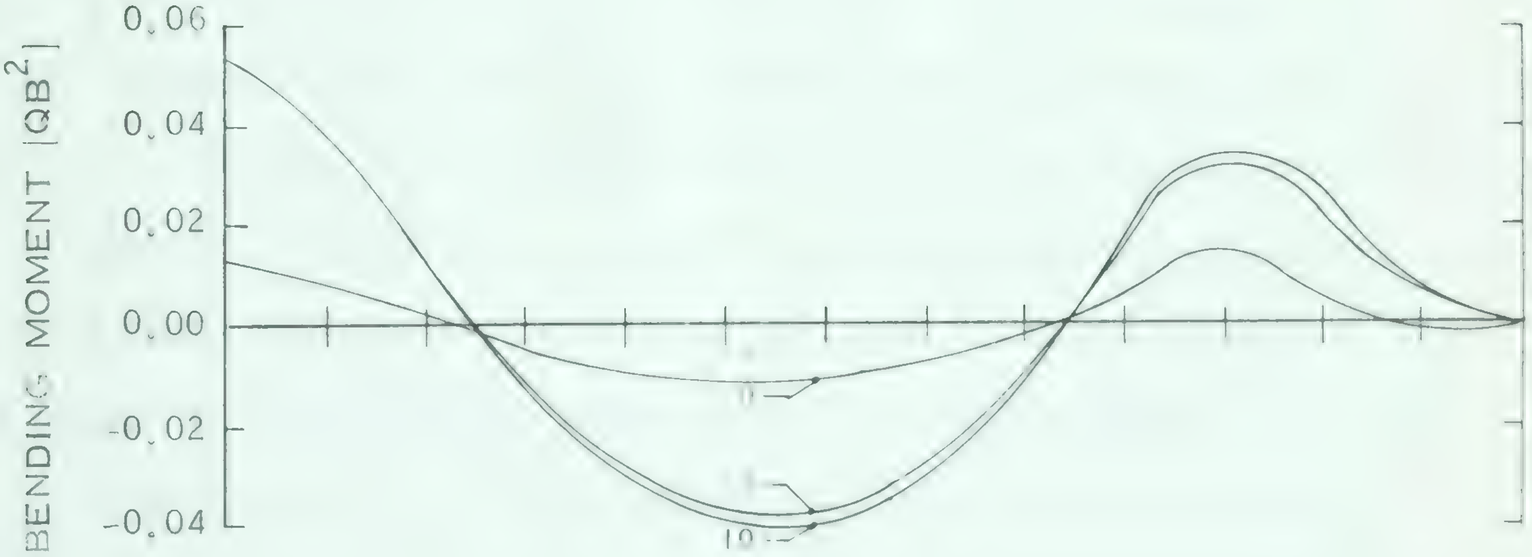


FIGURE 31 – SLAB BENDING MOMENTS ACROSS 13-X
UNIFORM LOAD, OH = 0.3, J = 1, K = 0

one beam must be carried as a torsional moment in the other. Associated with this moment transfer is the tendency for the beam carrying the torsion moment to rotate about its longitudinal axis in such a manner as to induce positive moment across the slab edge. This rotation increases the magnitude of the maximum positive moment and extends the positive moment portion of the slab as shown in FIGURE 31 for grid line 14-X when OH=0.4 and K=0. The positive region never reaches the corner, because at the intersection of 14-X with 14-Y the value of m_x is the negative beam moment divided by H. The opposite behavior occurs for overhang ratios smaller than 0.3.

Since the deflection patterns are almost unchanged, the effects of an increase in Poisson's ratio can be predicted qualitatively using the bending moments about the X-X and Y-Y axis for $\mu = 0$. If both these moments have the same sign at a particular point, the magnitude of each will be increased, but if the signs differ, their magnitudes will decrease. This is illustrated in the case of OH=0.3, H=J=0, and $K=\infty$ along the column lines where an increase in μ from 0 to 0.3 increases the maximum negative bending moments approximately 20% and decreases the maximum positive bending moment approximately 7%. Along grid lines midway between the columns the negative moments are reduced and the positive moments increased. The moments along grid lines in the cantilevered portion are affected very little by changes in Poisson's ratio.

Increasing Poisson's ratio has a substantial effect on the bending moments across grid lines near the edge when the perimeter beam stiffness is small. The positive humps in the bending moments near the edge in the vicinity of the column lines are reduced to such an extent that

positive moments are illiminated. There also exists a positive hump midway between the columns on which the smallest negative moment across the grid line occurs. The maximum negative bending moment is also reduced. For example, when OH=0.3, H=J=0, and K=∞ the moments across 12-X only range from -0.0085 qb^2 to -0.0018 qb^2 for μ = 0.3 compared to -0.0107 qb^2 to +0.0084 for μ = 0. However, when large beam stiffness exists this same behavior is observed, but to a much smaller extent. For example, when OH=0.3, H=10, J=1, and K=∞ the bending moments across 13-X range from -0.040 qb^2 to 0.059 qb^2 when μ =0.3 to from -0.041 qb^2 to 0.058 qb^2 when μ =0. In both cases of large beam stiffnesses the maximum positive moments occur at point (0,13).

4.4 Line Load Deflections

The deflection pattern resulting from an edge line load is not as sensitive to changes in the overhang ratio as that resulting from a uniform load. The fact that in all cases the edge deflects downward and the interior of the slab upward is illustrated in the diagonal deflections of FIGURE 32 for H=J=K=0. The deflections of the remainder of the slab can be estimated within reasonable limits from the extreme values occurring along the diagonal and the edge which are presented in FIGURES 32 and 33 respectively.

Increased values of the column bending stiffness, K, appreciably reduce the magnitude of, but not the variation in the edge deflections. Increasing K from zero to infinity decreases the deflection at the corner and the mid point 45.8% and 31.4%, 40.85% and 30.7%, 36.2% and 27.2%, and 32.4% and 24.1% for overhang ratios of 0.2, 0.3, 0.4, and 0.5 respectively. The maximum interior deflections are reduced approximately 35% for all

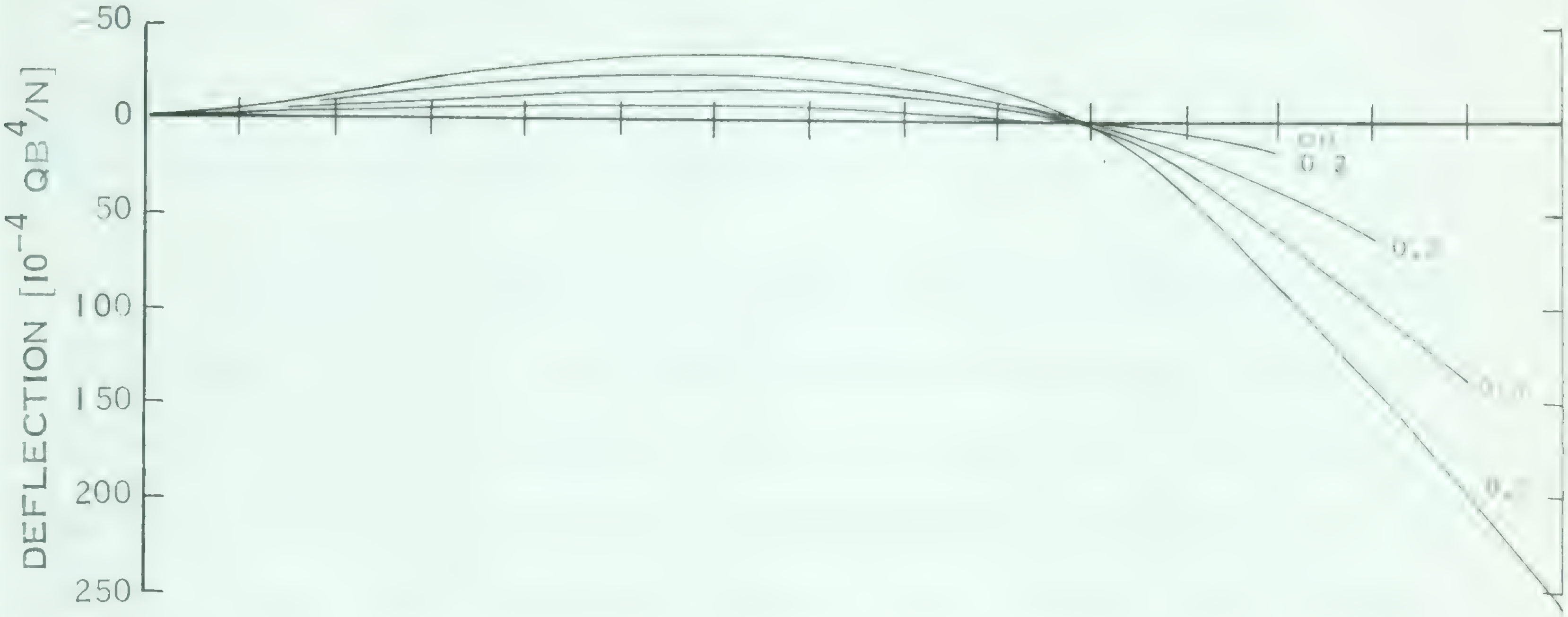


FIGURE 32 - SLAB DEFLECTIONS ALONG DIAGONAL LINE LOAD, H = 0, J = 0, K = 0

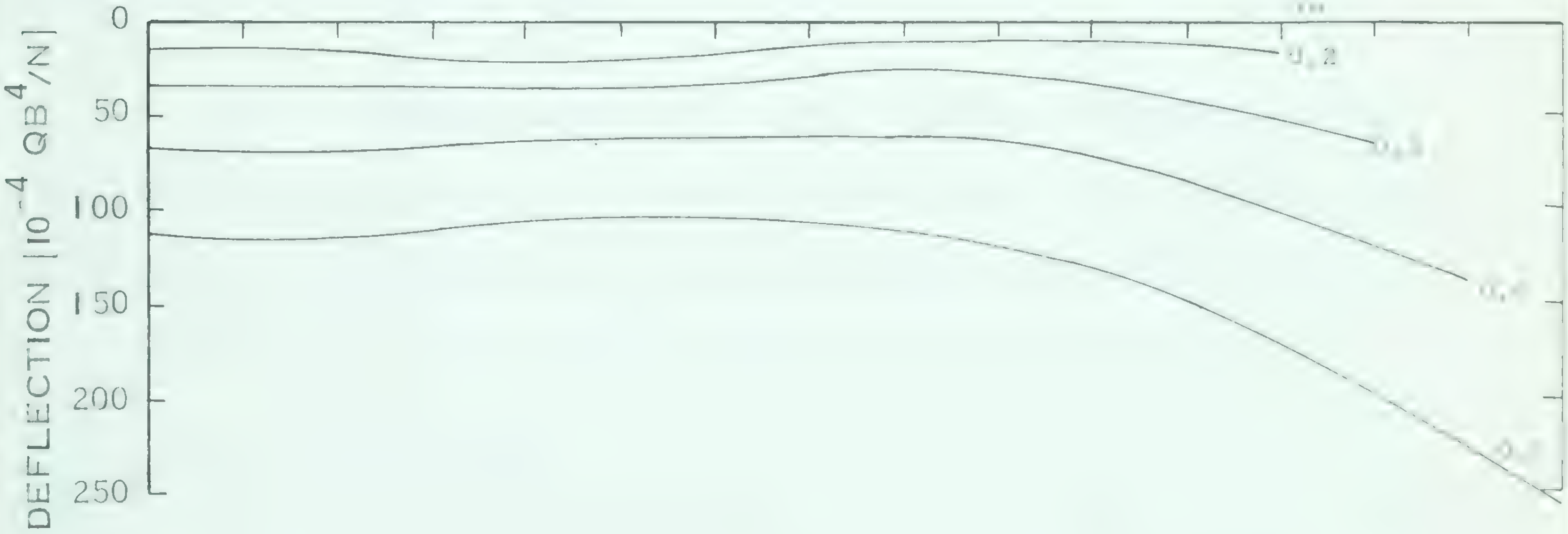


FIGURE 33 - SLAB DEFLECTIONS ALONG EDGE LINE LOAD, H = 0, J = 0, K = 0

overhang ratios. Intermediate values of column stiffness between zero and infinity result in deflections distributed between the extremes in a manner similar to that shown in FIGURE 13.

Increasing the edge beam bending and torsional stiffness reduced the magnitude of, and the variation in the edge deflections. For example, when K=0 an increase in stiffnesses from H=J=0 to H=10 and J=1 reduced the magnitude of the edge deflections at the corner and mid point by 46.4% and 66.0%, 50.5% and 66.6%, 47.6% and 65.1% and 43.0% and 64.4% for overhang ratios of 0.2, 0.3, and 0.4, and 0.5 respectively. The maximum interior deflections are reduced approximately 48%.

The combined effects on the deflections resulting from both increased beam and column stiffnesses are not directly additive, but similar to that occurring with a uniform load.

For an overhang ratio of 0.3, an increase in Poisson's ratio from 0 to 0.3 has a small effect on the deflections. For example, when J=H=0 and $K=\infty$ the maximum deflection was decreased 12.37% and when H=10, J=1, $K=\infty$ the maximum deflection was decreased 1.63%.

4.5 Line Load Bending Moments

When the value of the column and beam stiffnesses are zero the shape of the m_x diagrams along grid lines parallel to the X-X axis can be visualized by considering analogous continuous beam strips simply supported over the column lines. For interior grid lines near 0-X the shape of the bending moment digrams are similar to those of the analogous beams loaded with a point load at each end, whereas for grid lines in the region of

the columns adjacent to the cantilevered portion the shape approaches that obtained with a uniform load over the entire length. This is evident in FIGURES 34 to 37 which show the moments along grid lines 0-X, 5-X, 10-X and the edge for H = J = K = 0. The distribution of the moments along the grid lines between these values can be interpolated linearly without serious error. The slab twisting stiffness results in positive humps on the column lines, similar to, but much smaller than those exhibited with the uniform load near the edge and inside the columns. However, these do not become positive unless edge beams are present.

Increasing the column bending stiffness has a much more widespread effect on the moment with the line load than it has for the uniform load. Infinitely stiff column reduce the maximum bending moments in the interior panels from 40% to 45%. FIGURE 38 shows this reduction along 0-X for OH=0.5 and H=J=0 as the column stiffness increases from zero to infinity.

It was noted that at the inside face of the edge columns with bending stiffness the bending moment is positive. This is due to the tendancy of the slab over the column to slope towards the edge. The stiff column resists this slab rotation resulting in compression on the top of the slab across 10-X and compression on the bottom across 10'-X. This behavior is exhibited for all overhang ratios and finite values of column stiffness at the edge columns. Similar positive moments are present at the interior face of the corner columns for all overhang ratios larger than approximately 0.25. For H=J=0 the magnitude of these positive moments adjacent to the infinitely stiff edge and the corner columns are 0.31 and -0.12, 0.54 and 0.10, 0.78 and 0.32, and 1.02 and 0.56 pb for overhang

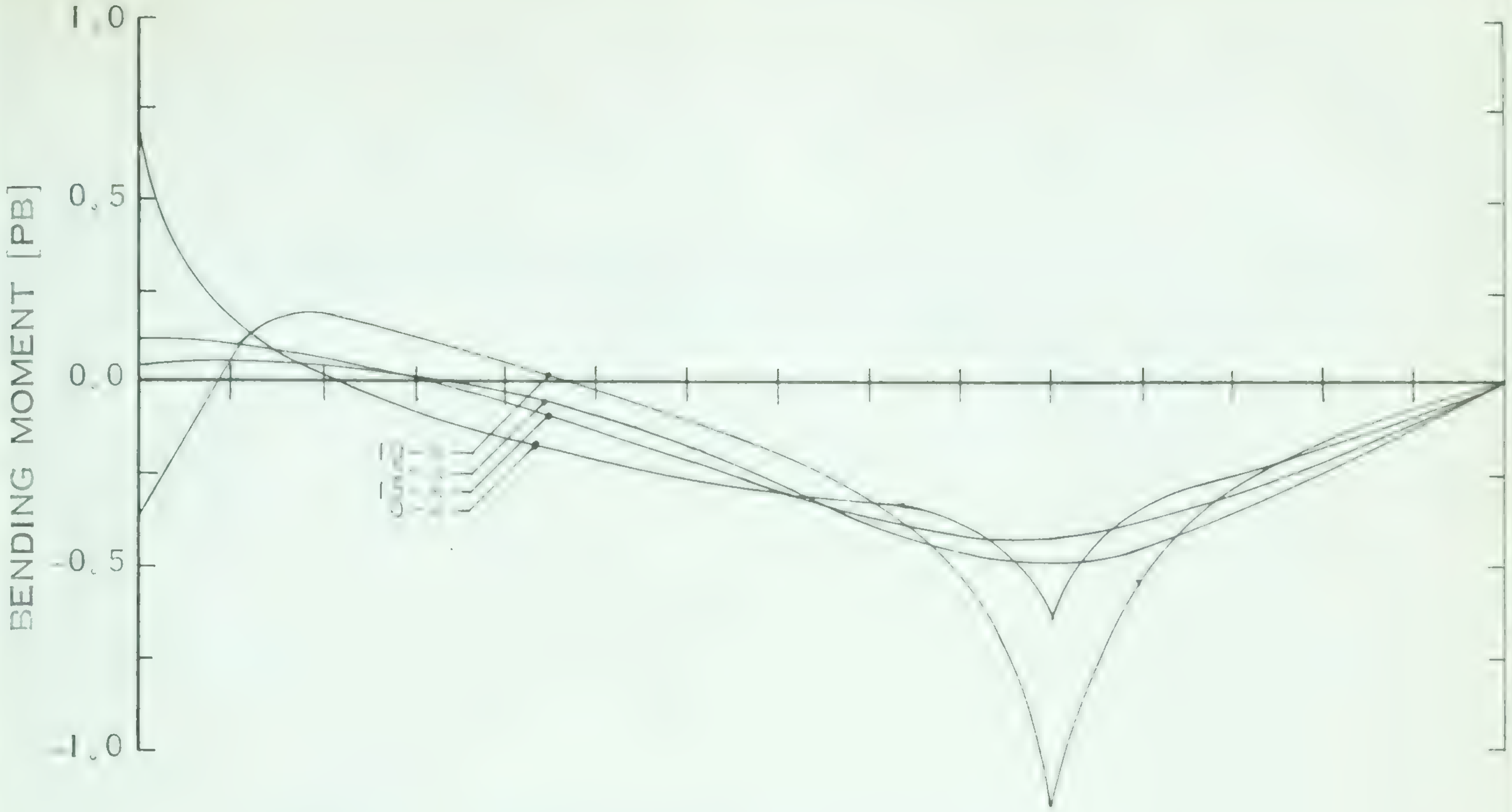


FIGURE 34 - SLAB BENDING MOMENTS
LINE LOAD. OH = 0.5. H = 0. J = 0, K = 0

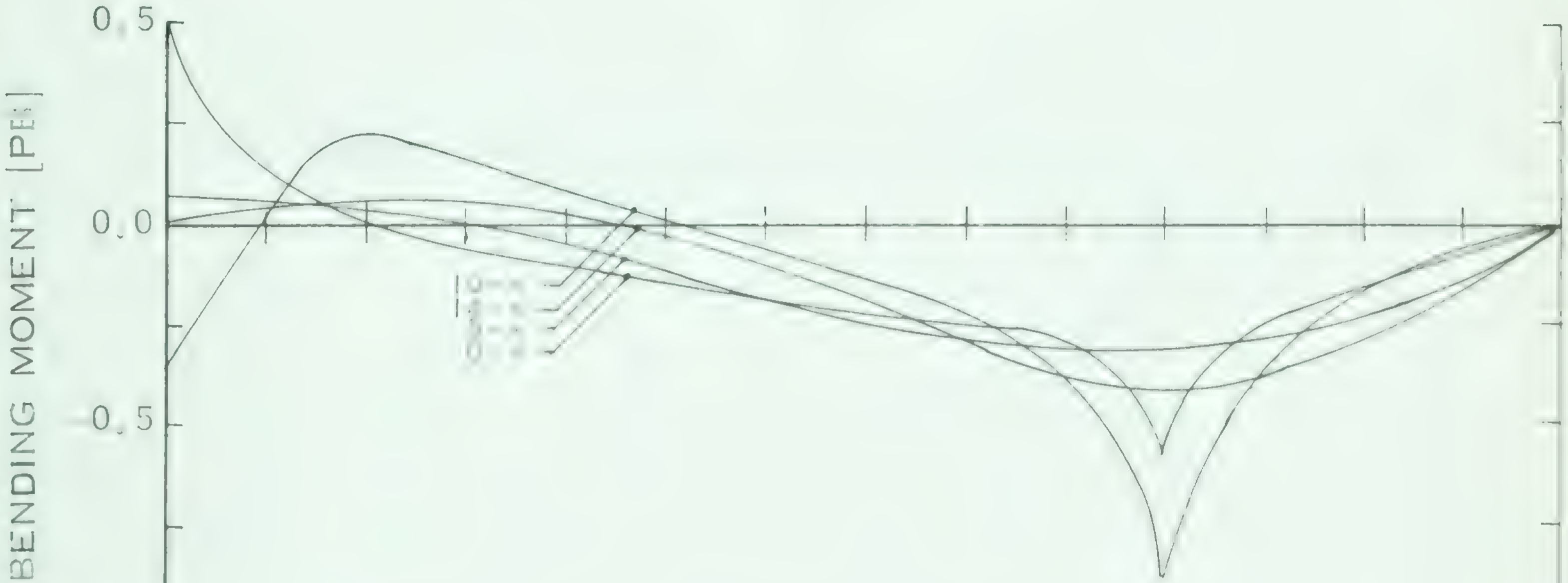


FIGURE 35 - SLAB BENDING MOMENTS
LINE LOAD. OH = 0.4. H = 0. J = 0. K = 0

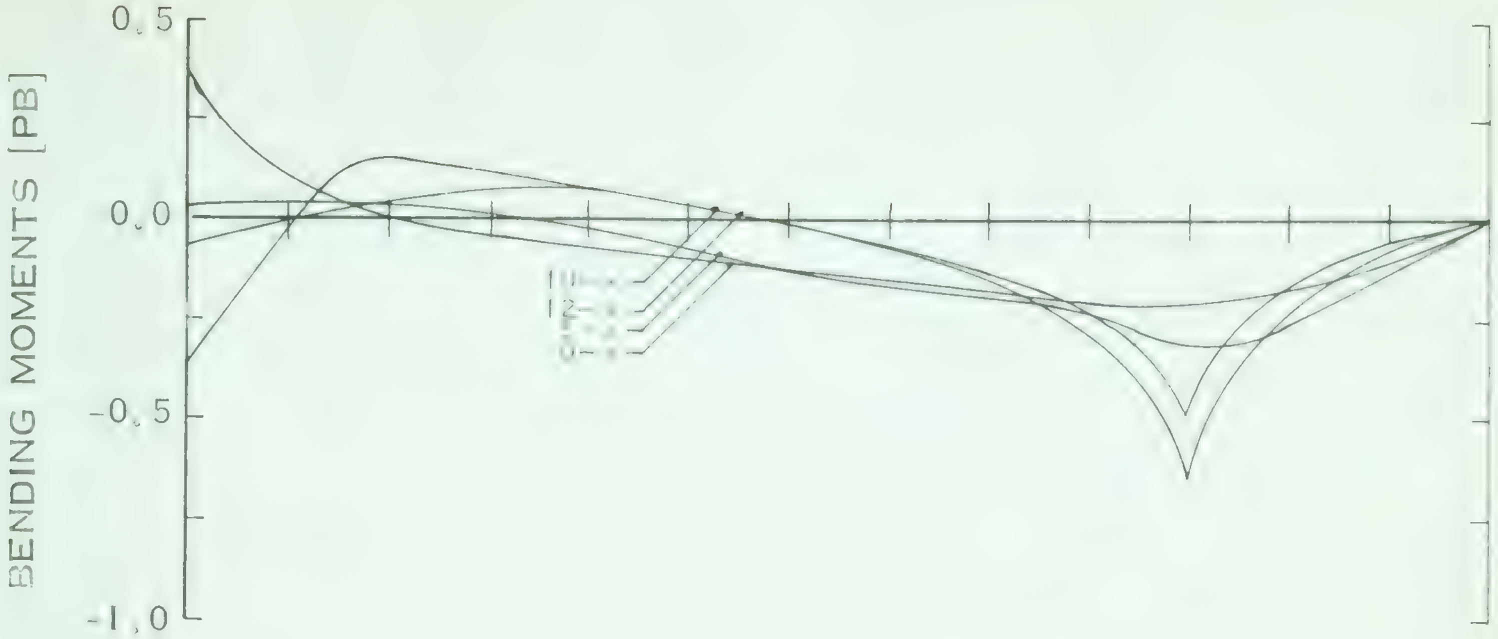


FIGURE 36 – SLAB BENDING MOMENTS
LINE LOAD, OH = 0.3, H = 0, J = 0, K = 0

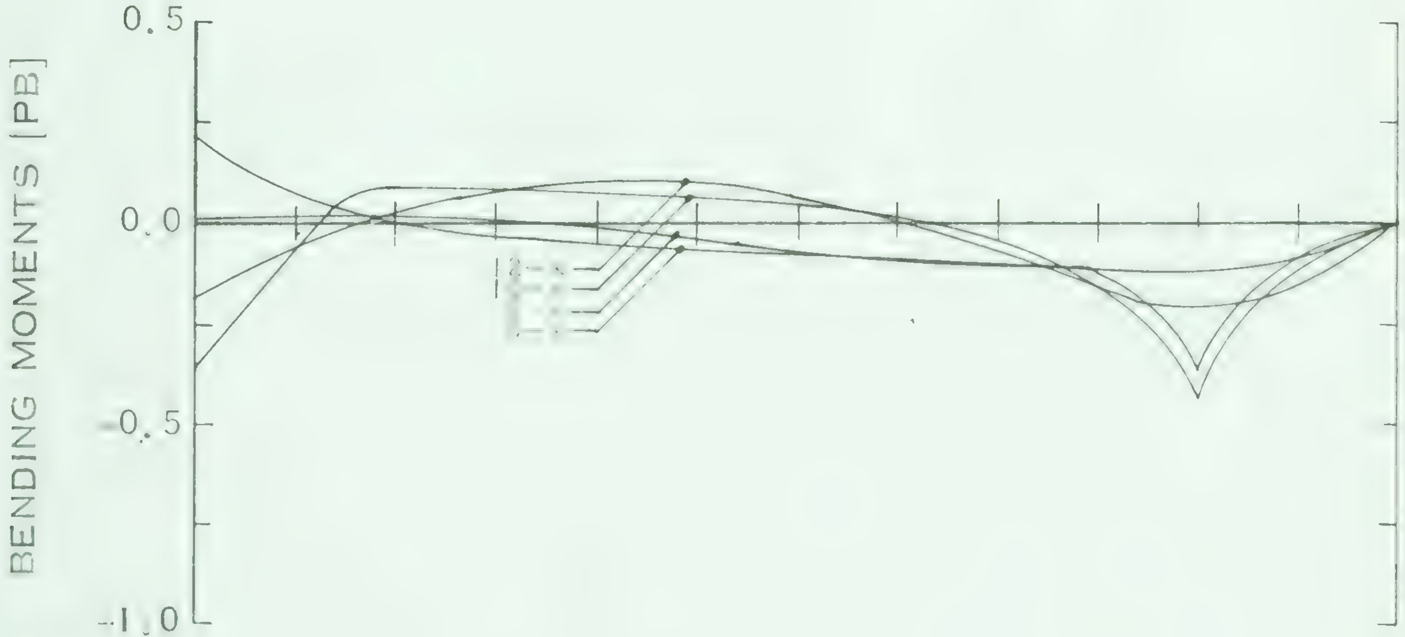


FIGURE 37 – SLAB BENDING MOMENTS
LINE LOAD, OH = 0.2, H = 0, J = 0, K = 0

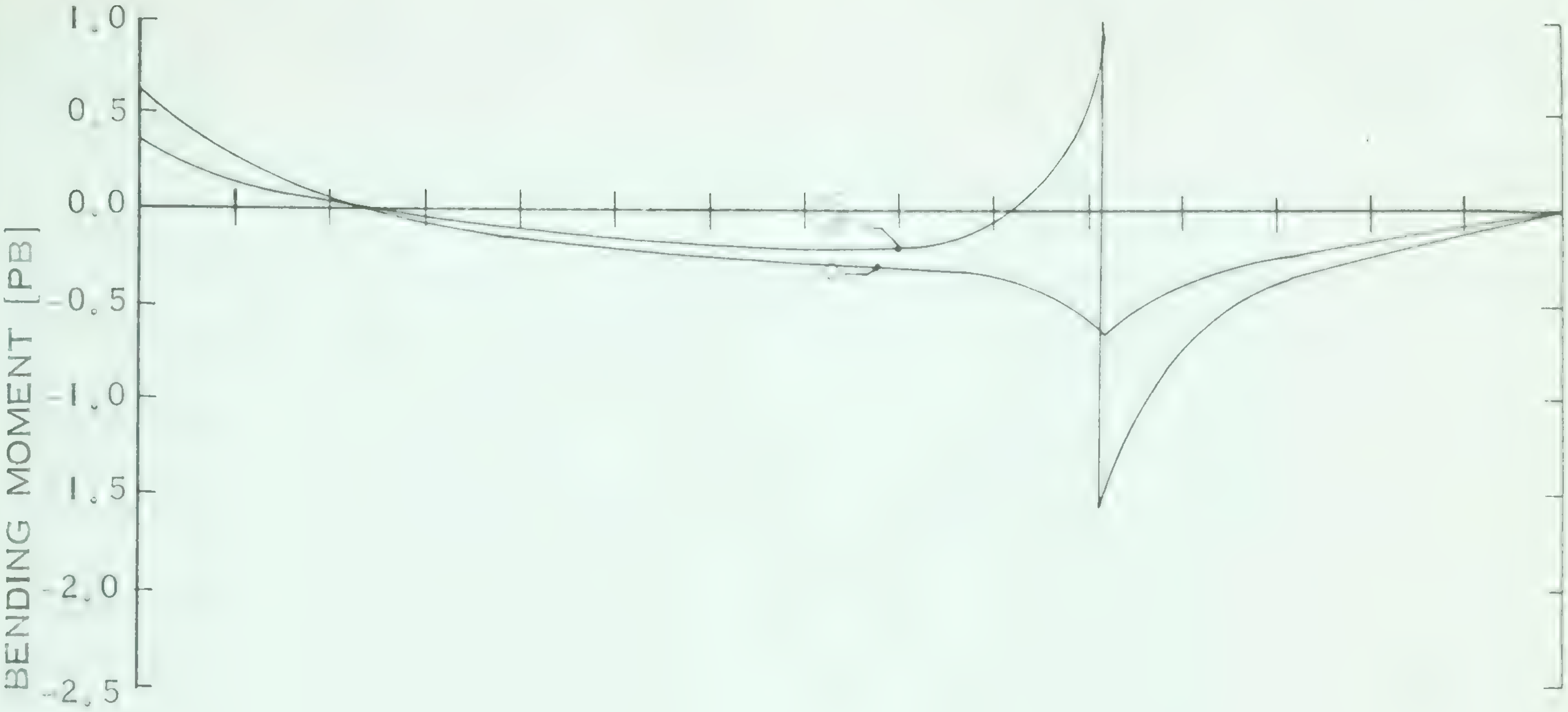


FIGURE 38 - SLAB BENDING MOMENTS ALONG 0-X
LINE LOAD, OH = 0.5, H = 0, J = 0

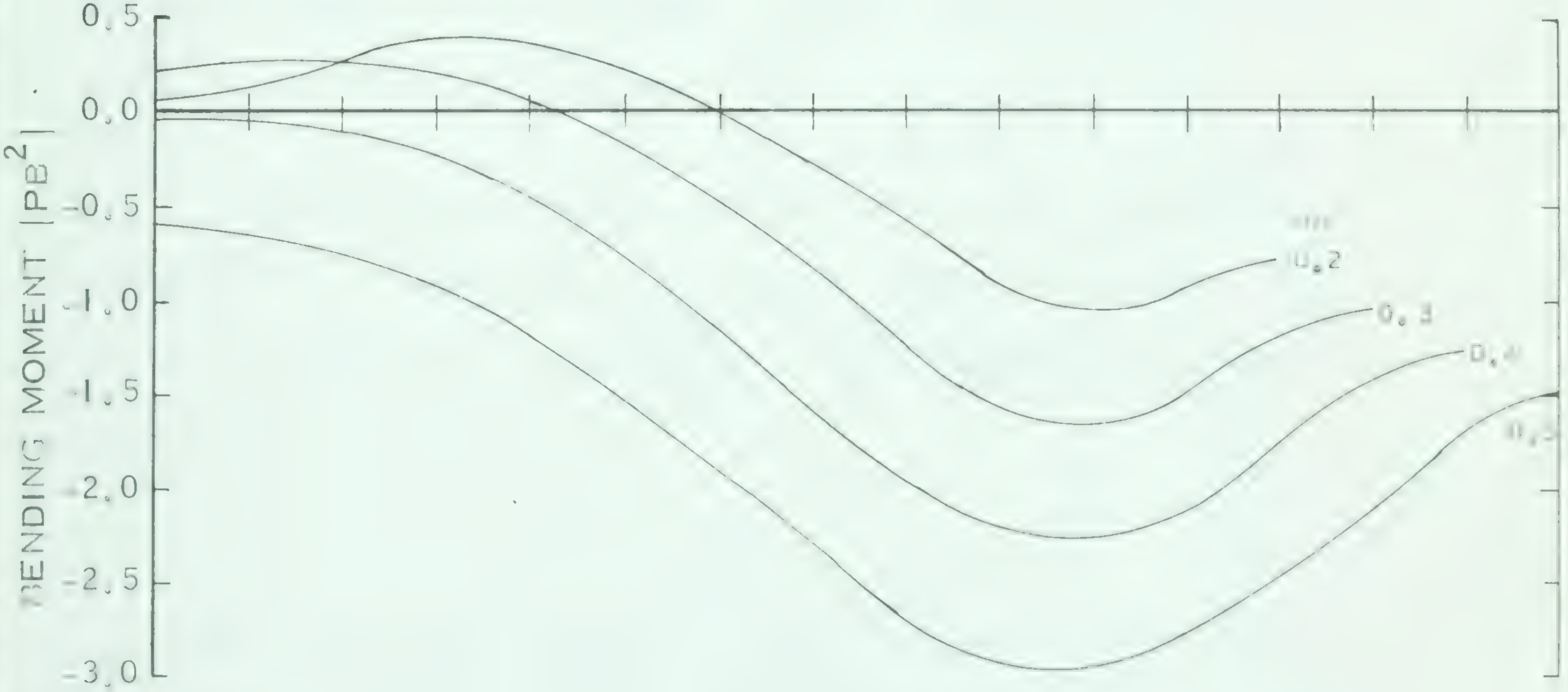


FIGURE 39 - BEAM BENDING MOMENTS
LINE LOAD, H = 3, J = I, K = 0

ratios of 0.2, 0.3, 0.4 and 0.5 respectively.

Increasing the edge beam bending and torsional stiffness reduces slab bending moments in the interior panels. For example: an increase in the beam stiffnesses from H=J=0 to H=10 and J=1 reduces these moments approximately the same amount as changing the column stiffness from zero to infinity. The bending moments across the slab edge near the corner column are positive when the stiffening of the edge beams are large. When H=10, J=1 and K=0 the maximum positive edge moments of 0.26, 0.30, 0.33 and 0.35 pb for overhang ratios of 0.2, 0.3, 0.4, and 0.5 respectively are in the same order of magnitude as the maximum positive interior bending moments. Infinitely stiff columns reduce these values approximately 11%. The positive edge moments are reduced to very small negative values midway between the columns and at the corner. Stiff edge beams produce very small moments across the edge near 0-Y which are positive for small and negative for large over-hang ratios.

The edge beam bending and torsional moments are presented in FIGURES 39 and 40 respectively for values of H=3, J=1 and K=0. Decreasing H and/or J and/or increasing K reduces the value but not the distribution of these moments.

Since the deflection pattern is basically the same for all values of Poisson's ratio, the qualitative effects of increases in Poisson's ratio can be predicted from the deflections occurring when $\mu = 0$ and the definition of bending moment in identically the same manner as with the uniformly distributed load.

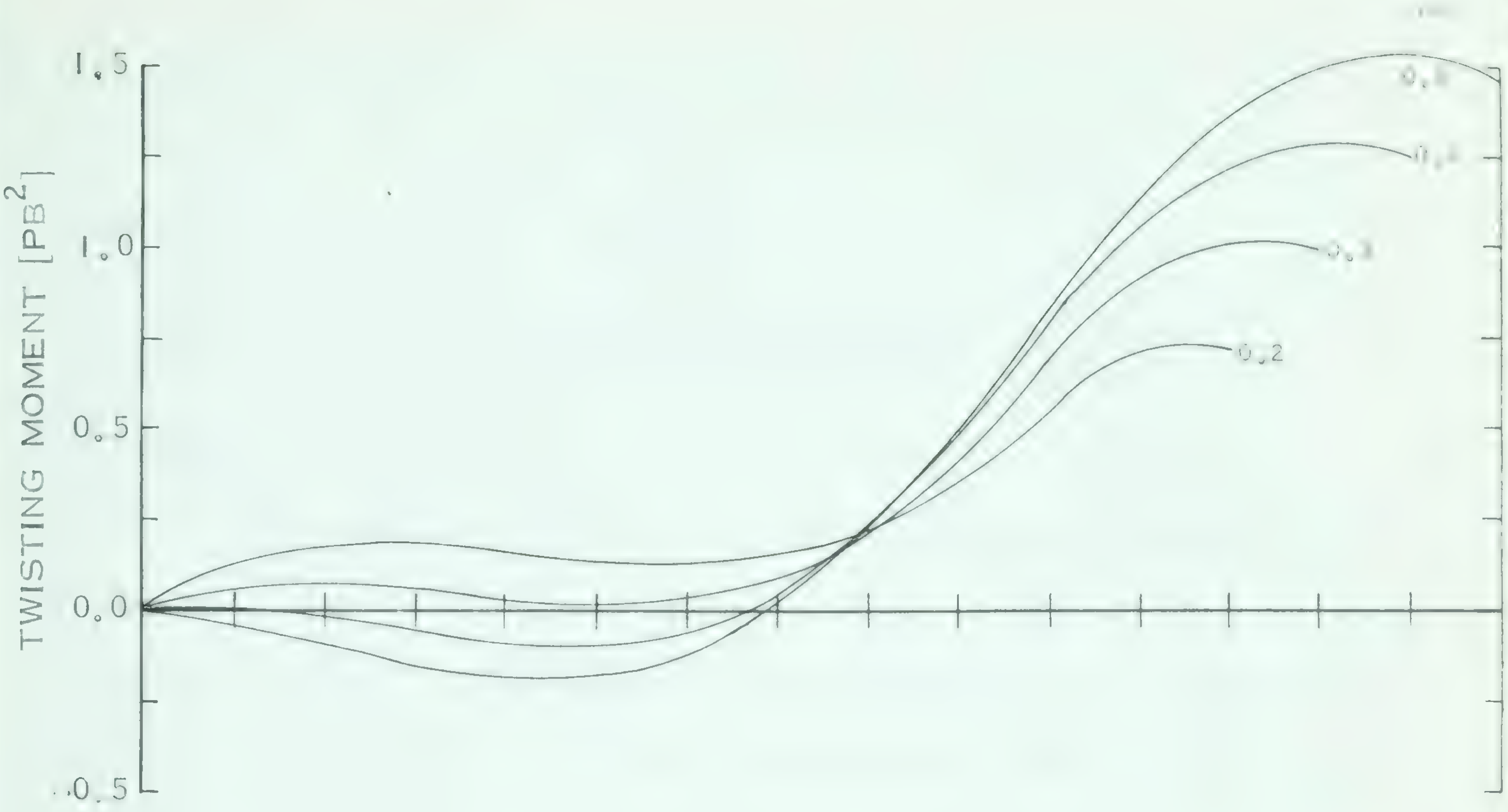


FIGURE 40 – BEAM TWISTING MOMENTS
LINE LOAD, H = 3, J = 1, K = 0

CHAPTER V

SUMMARY AND CONCLUSIONS

5.1 Summary

This investigation examined the effects on the behavior of flat plate cantilevered slabs resulting from changes in some of the pertinent variables. A total of 124 combinations of overhang ratio, column bending stiffness, beam bending and torsional stiffnesses, Poisson's ratio, and loading were considered.

The structure analysed, FIGURE 1, consists of a four panel square slab, with equal overhangs on all four sides, supported on nine non-settling columns of variable bending stiffness. The slab edges were stiffened with beams processing variable bending and torsional stiffnesses. A uniformily distributed load over the entire slab and a uniform line load along the entire edge were considered for each combination of the other variables.

The problem was solved by a numerical procedure based on Newmark's plate analogy. The resulting simultaneous equations were solved by a method which takes full advantage of the matrix population scarcity.

5.2 Conclusions

The following are conclusions drawn from the analytical results of the flate plate cantilevered slab investigated.

(a) Deflections

(1) The general deflection pattern of the slab may be

predicted by considering continuous beam strips simply supported over the column lines. However, the numerical values of the maximum deflections obtained in this manner may be 100% in error.

(2) The overhang ratio is the most important variable affecting the deflections. For a uniformily distributed load, overhang ratios larger than approximately 0.4 result in all of the slab cantilevered portion deflecting downward. For smaller overhang ratios part of the cantilevered portion rises.

(3) Increasing the edge beam and column bending stiffnesses reduces the maximum deflection.

(4) Increasing the edge beam bending stiffness reduces the magnitude and variation in the edge deflections.

(5) The deflection of the slab corner is more sensitive to changes in the variables than any other location on the slab. For example, when K=H=J=0 and a uniform load acts over the entire slab, an increase in the overhang ratio from 0.2 to 0.5 result in change in the corner deflection from 0.0052 qb^4/N upward to 0.0295 qb^4/N downward, while the maximum interior deflection changes only from 0.0094 qb^4/N downward to 0.0035 qb^4/N downward.

(6) Poisson's ratio has a very small effect on the distribution and magnitude of the deflections.

(7) The effect on the deflections of any particular variable depends on the numerical values of the remaining variables. For this reason, the effect of changing more than one variable at a time can not be predicted by directly adding the effects of each individually.

(b) Bending Moments

(1) The bending moment is not constant across grid lines

in the cantilevered portion of the slab.

(2) Positive bending moments exist near the edge of the cantilevered portion in the region of the column lines for all the cases of uniform load considered. These moments are of the same order of magnitude as the maximum positive interior moments when the edge beam stiffness is large. The line load bending moments also become less negative in these regions, but only become positive when the edge beam stiffness is large.

(3) The maximum slab bending moments are localized to the immediate vicinity of the columns.

(4) Changes in the value of the column bending stiffness affects the slab bending moments to an appreciable extent only in the vicinity of the columns for the uniformily distributed load, but over the entire slab for a perimeter line load.

(5) Increasing the edge beam bending stiffness does not appreciably affect the distribution of the total slab moment across the grid lines except near the edge beams where the moments are reduced.

(6) Increasing the edge beam bending stiffness reduces the magnitude of the total moment carried by the slab across all grid lines.

(7) Increasing the edge beam torsional stiffness significantly increases the positive bending moments at the slab edges.

(8) Highly localized positive bending moments exist at the inside faces of the edge and corner columns when the slab is loaded with a perimeter line load.

(9) Qualitatively the effects on the bending moments

in the slab for other values of Poisson's ratio can be predicted from the slab curvatures for $\mu = 0$. Due to the elastic support conditions the quantitative changes can not be computed by the method usually used for plates with fixed or hinged boundary conditions.

(10) The effects of all the variables controlling the bending moments in the slab are interrelated and therefore individual effects are not directly additive.

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APPENDIX A

FINITE DIFFERENCE PATTERNS FOR MOMENTS

Slab

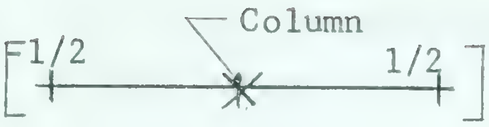
$$M_{ox}=\frac{N}{h}\begin{bmatrix} & -\mu & \\ -1 & 2+2\mu & -1 \\ & -\mu & \end{bmatrix} w \qquad M_{oy}=\frac{N}{h}\begin{bmatrix} & -1 & \\ -\mu & 2+2\mu & -\mu \\ & -1 & \end{bmatrix} w$$

$$M^{B}{}_{xy}=\frac{N}{h}\begin{bmatrix} 1-\mu & -1+\mu \\ -1+\mu & 1-\mu \end{bmatrix} w \qquad M^{A}{}_{xy}=\frac{N}{h}\begin{bmatrix} 1-\mu & -1+\mu \\ -1+\mu & 1\ \text{[illegible]} \end{bmatrix} w$$

$$M^{C}{}_{xy}=\frac{N}{h}\begin{bmatrix} 1-\mu & -1+\mu \\ -1+\mu & 1+\mu \end{bmatrix} w \qquad M^{D}{}_{xy}=\frac{N}{h}\begin{bmatrix} 1-\mu & -1+\mu \\ -1+\mu & 1+\mu \end{bmatrix} w$$

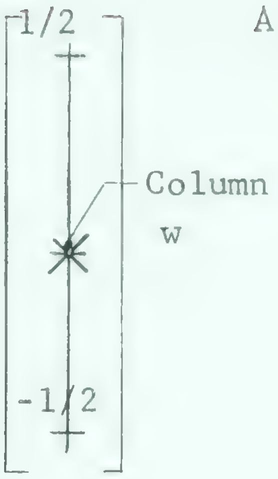
Column

$M_{cx}=\frac{KN}{h}$



w

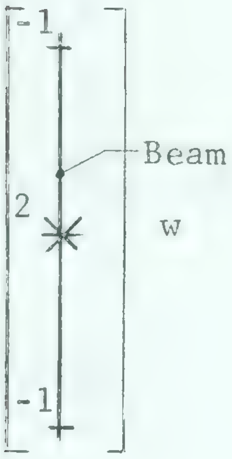
$M_{cy}=\frac{KN}{h}$



w

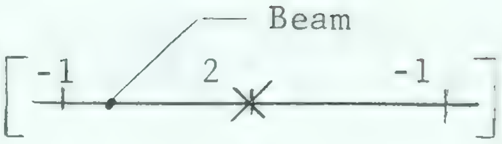
Beam

$M_{oy}=\frac{NH}{h}$



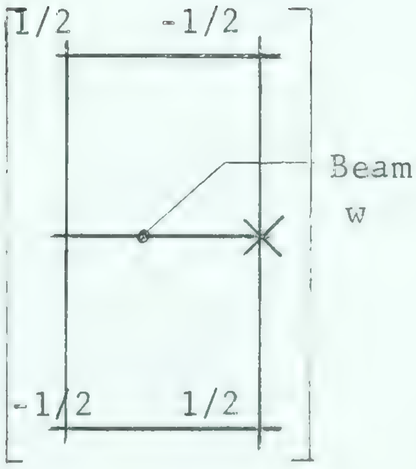
w

$M_{ox}=\frac{NH}{h}$



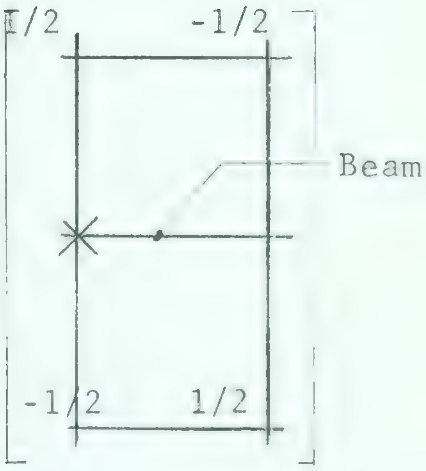
w

$T_{ow}=\frac{JN}{h}$

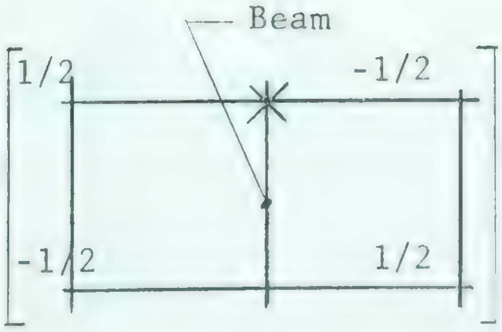


w

$T_{oe}=\frac{JN}{h}$

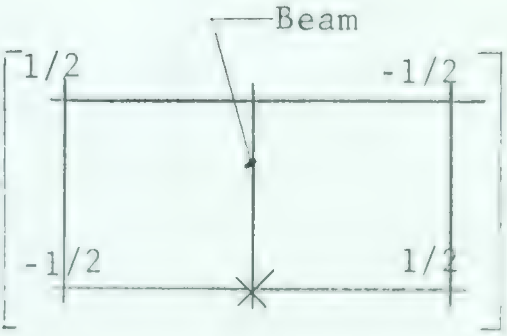


$T_{os}=\frac{JN}{h}$



w

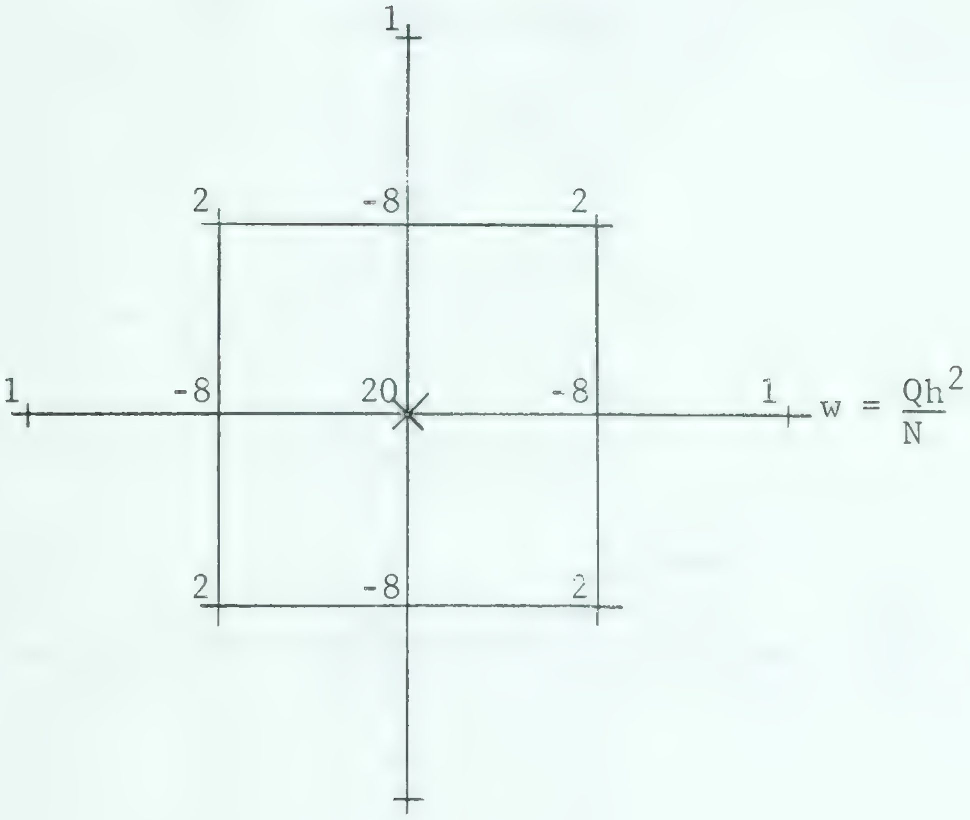
$T_{on}=\frac{JN}{h}$

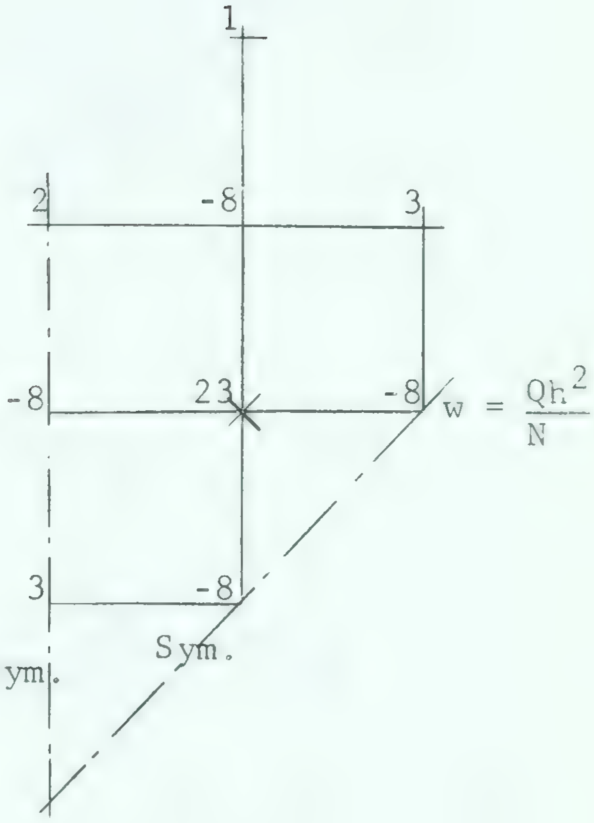


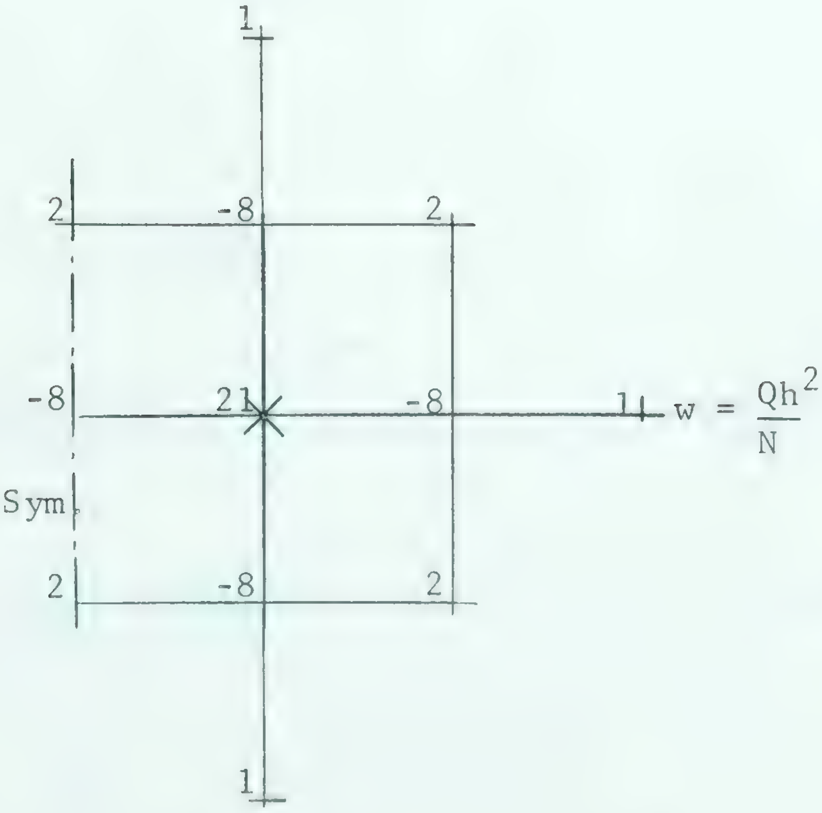
w

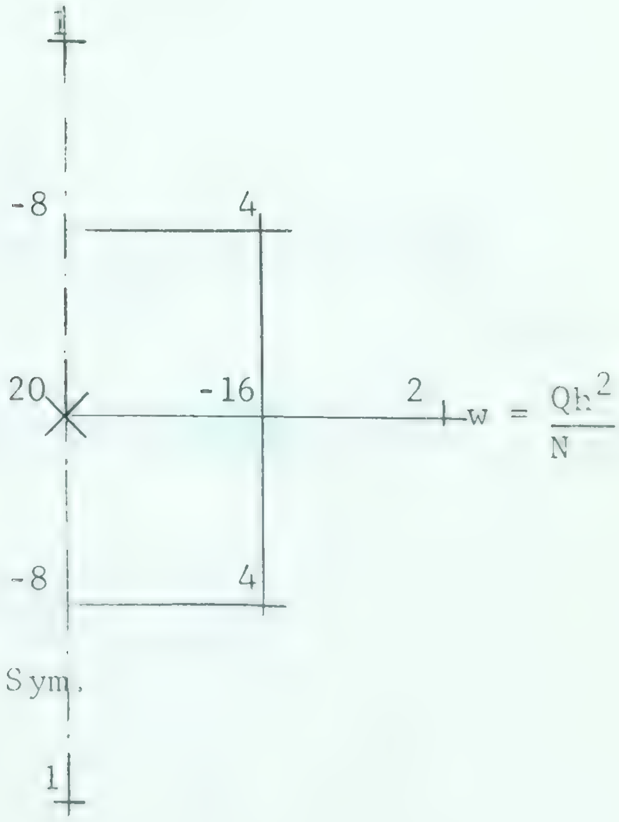
APPENDIX B

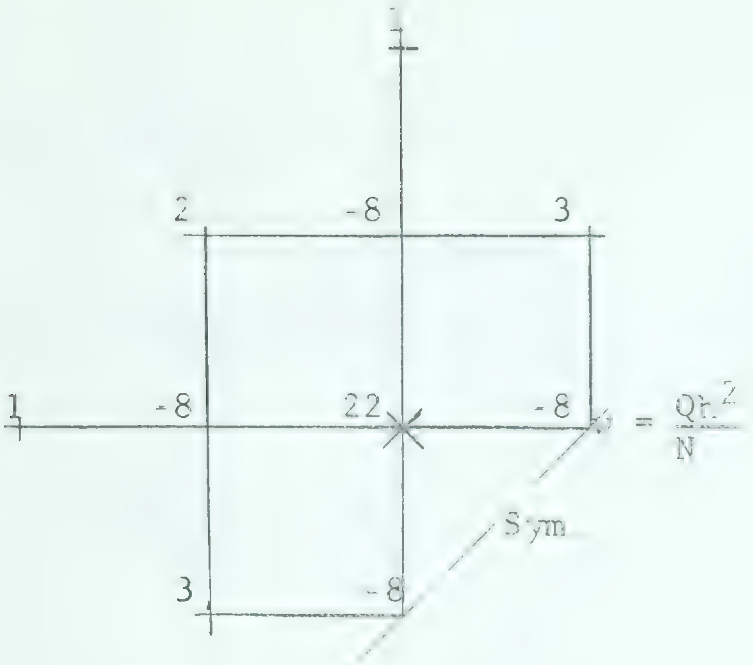
OPERATORS

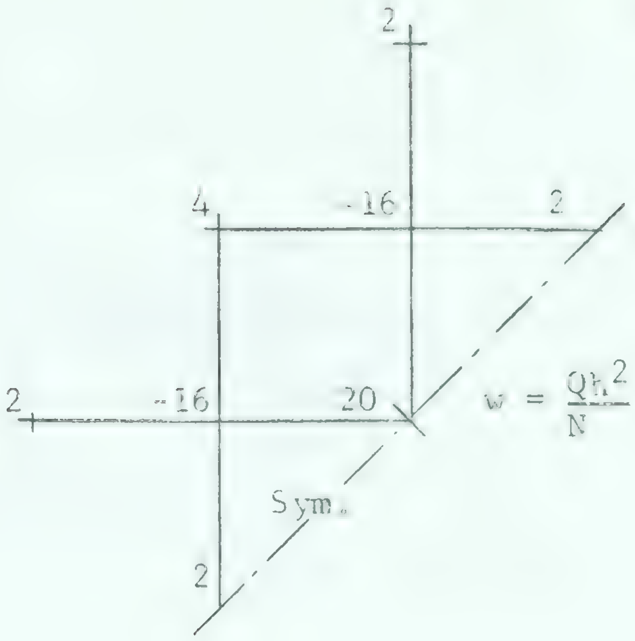




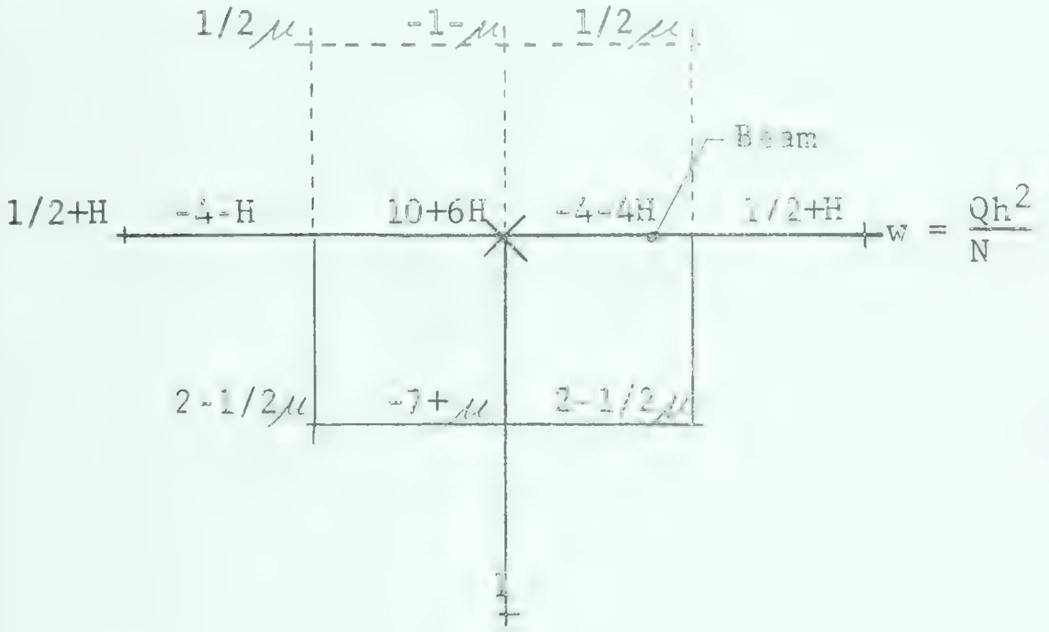


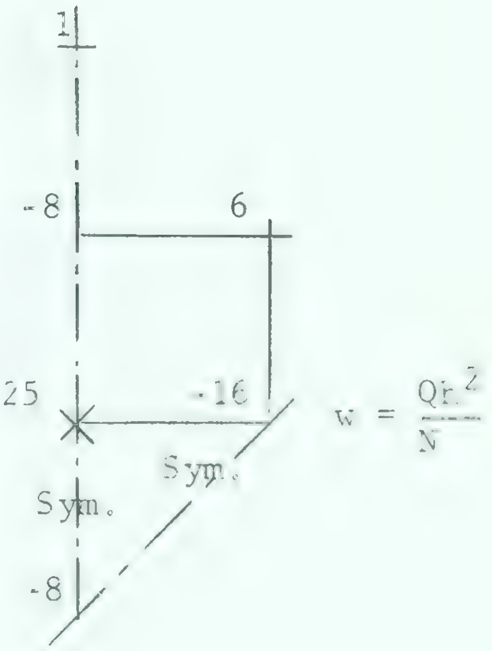


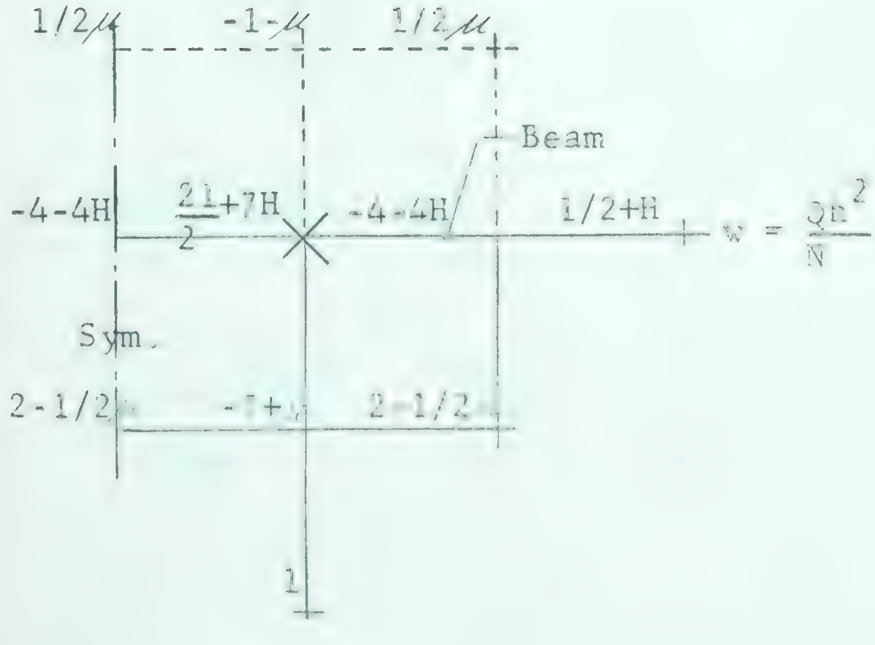


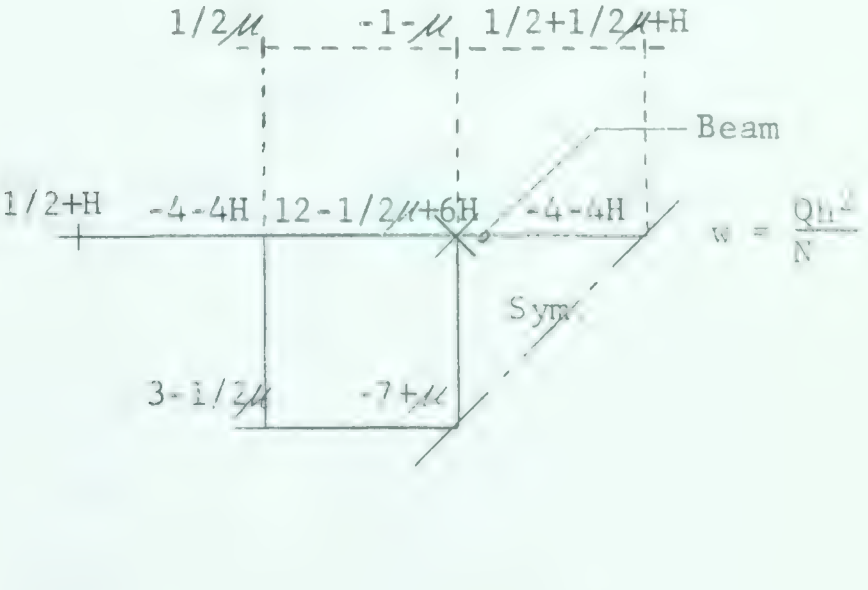


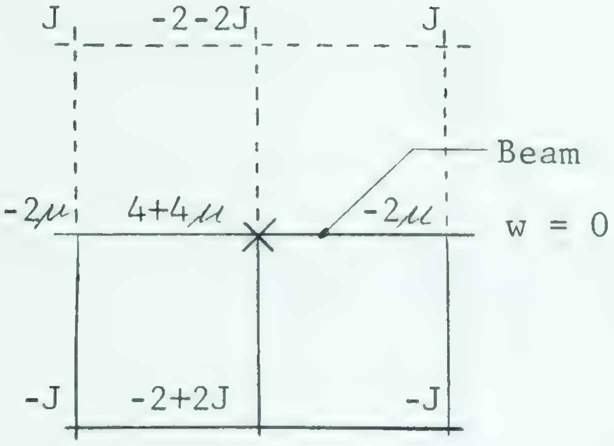
Do not use near columns.

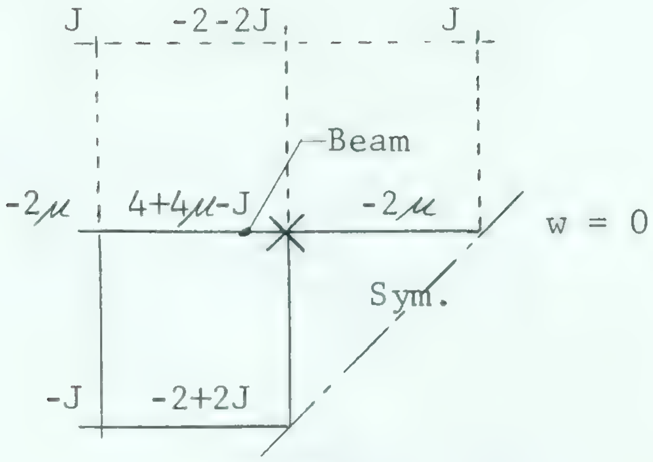


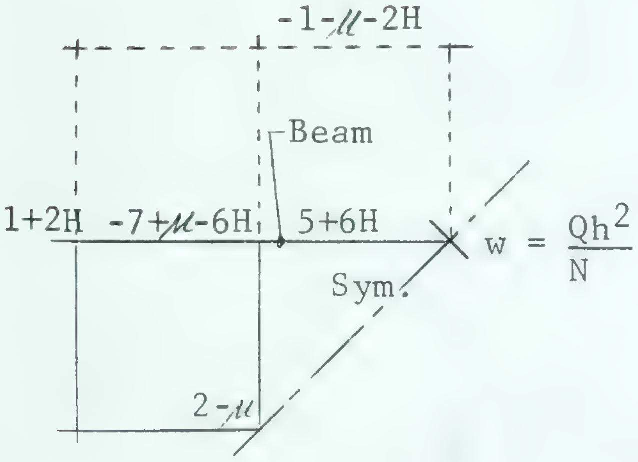


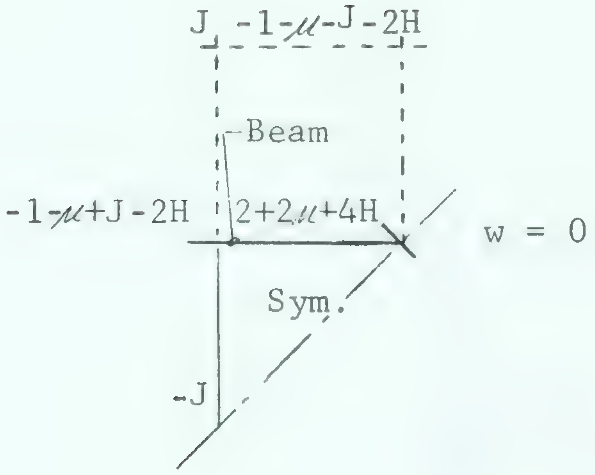


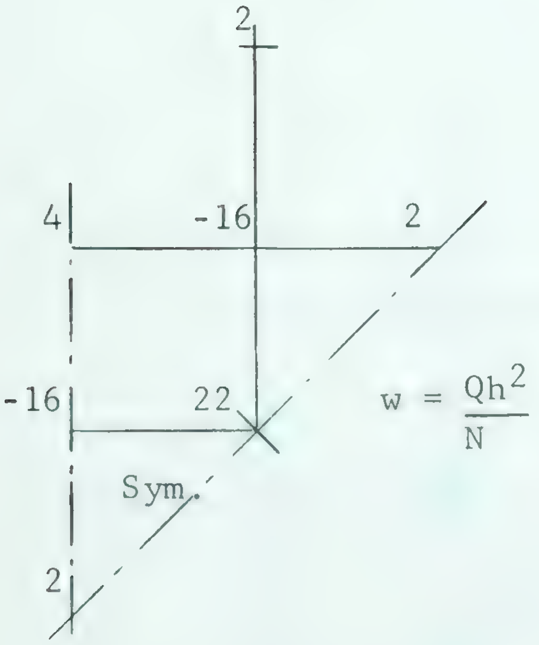


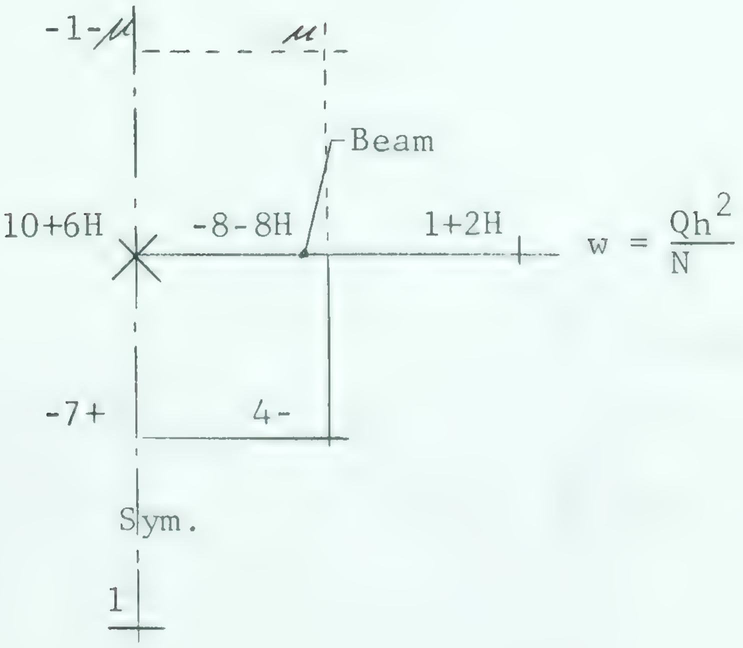

J
-2-2J
J
Beam
-2μ
4+4μ
-2μ
w = 0
-J
-2+2J
-J

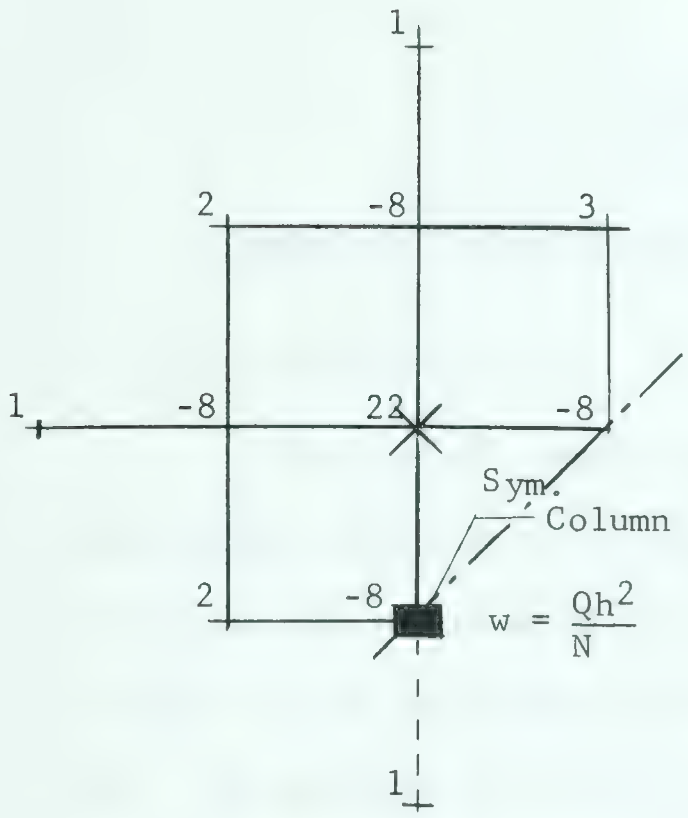

J
-2-2J
J
Beam
-2μ
4+4μ-J
-2μ
w = 0
Sym.
-J
-2+2J

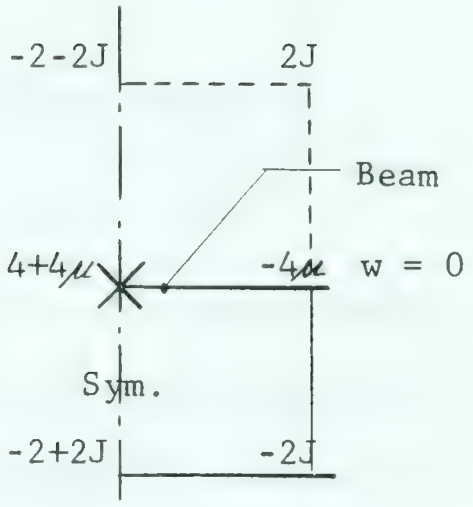

-1-μ-2H
Beam
1+2H
-7+μ-6H
5+6H
$w = \frac{Qh^2}{N}$
Sym.
2-μ

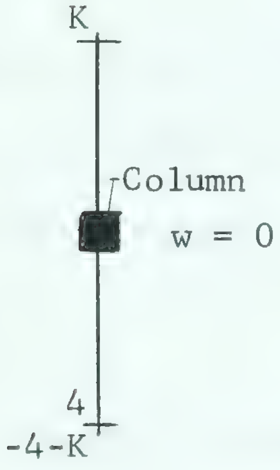

J
-1-μ-J-2H
Beam
-1-μ+J-2H
2+2μ+4H
w = 0
Sym.
-J

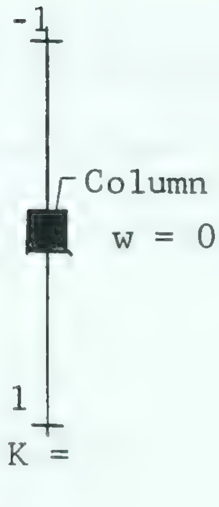

2
4
-16
2
-16
22
$w = \frac{Qh^2}{N}$
Sym.
2

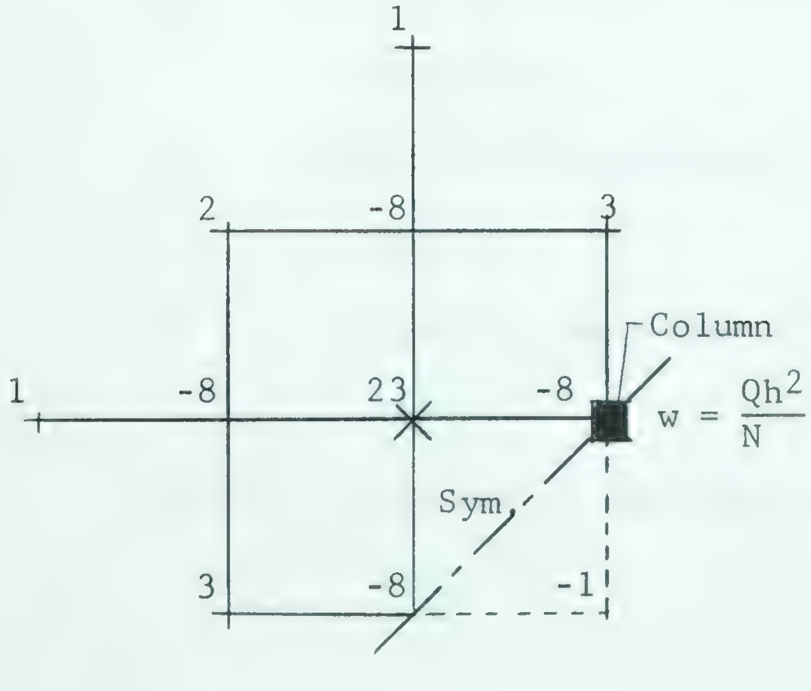

-1-μ
μ
Beam
10+6H
-8-8H
1+2H
$w = \frac{Qh^2}{N}$
-7+
4-
Sym.
1

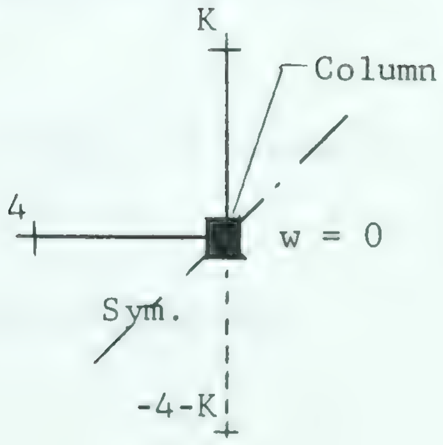

1
2
-8
3
1
-8
22
-8
Sym.
Column
2
-8
w = $\frac{Qh^2}{N}$
1


-2-2J
2J
Beam
4+4μ
-4α
w = 0
Sym.
-2+2J
-2J


K
Column
w = 0
4
-4-K


-1
Column
w = 0
1
K =


1
2
-8
3
Column
1
-8
23
-8
w = $\frac{Qh^2}{N}$
Sym.
3
-8
-1


K
Column
4
w = 0
Sym.
-4-K

APPENDIX C

Computer Program to Solve Finite Difference Equations

C.1 Introduction

The digital computer program comprising SECTION C4 solves the simultaneous equations generated from finite difference operators. The program is set up specifically to solve the equations resulting from a triangular grid but changes can easily be made if other grid systems are used. The program has solved 151 simultaneous equations in less than two minutes on an IBM 7090 computer.

The program, written in Fortran II, used the procedures outlined in SECTION 2.6 to solve the equations. Comment statements have been added to enable easier tracing to the steps used in arriving at the solution.

C.2 Notation

N = number of simultaneous equations to be solved

NUM = number of solutions or loadings being solved

MD = distance between extreme diagonal vectors or vector extensions

M = distance between main diagonal and extreme lower left diagonal

K = number of equations to first discontinuity of diagonal vectors

A(I,J) = elements of the matrix referred to using diagonal

subscripting. Dimension I for N and J for MD plus NUM.

ID(J) = distance diagonal vector J is from main diagonal measured positive to the left and/or down. Dimension for the number of diagonal vectors.

B(J) = element on the vector J for the equation being considered. Dimension for the number of diagonal vectors plus NUM.

C.3 Sample Problem

The two sets of equations shown in FIGURE 41 are similar in type to the finite difference equations generated from a triangular plate except that there are only three diagonal vectors. These will be solved to illustrate the workings of the program.

The input data and solution shown in SECTION C.4 are self explanatory. The three intermediate printouts of the matrix are not usually printed. They are included to make it easier to follow the steps of the Gauss-Elimination method employed.

C.4 Program and Sample Problem

$$\begin{bmatrix} 5 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 6 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 10 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & -8 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 11 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 13 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & -7 & 0 & 0 & 0 & 0 & 7 \end{bmatrix} \times \begin{bmatrix} X_1 & Y_1 \\ X_2 & Y_2 \\ X_3 & Y_3 \\ X_4 & Y_4 \\ X_5 & Y_5 \\ X_6 & Y_6 \\ X_7 & Y_7 \\ X_8 & Y_8 \\ X_9 & Y_9 \\ X_{10} & Y_{10} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1/2 \\ 1 & 1/2 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1/2 & 1 \\ 1/2 & 1/2 \end{bmatrix}$$

FIGURE 41 Equations for Sample Problem

```
..I FINITE DIFFERENCE SOLUTION 920134  PLEASE SAVE OUTPUT
..LOAD FORTRAN EXECUTE
      DIMENSION A(10,12),ID(3),B(5)
C READING PROPERTIES OF EQUATIONS
 1    READ INPUT TAPE 5,2
 2    FORMAT(51H                                                  )
      WRITE OUTPUT TAPE 6,2
      READ INPUT TAPE 5,3,N,NUM,MD,M,K
      READ INPUT TAPE 5,6,(ID(J),J=1, 3)
 3    FORMAT(5I10)
      LM= 3+NUM
 6    FORMAT( 3I5)
 4    FORMAT(1X,10F7.2)
 75   LEN=MD+NUM
      M1=M-1
C MAKING COMPLETE MATRIX ZERO
      DO 5 I=1,N
      DO 5 J=1,LEN
 5    A(I,J)=0.0
C READING AND PLACING VECTOR ELEMENTS IN PROPER LOCATIONS
      KI=K
      KX=-1
      WRITE OUTPUT TAPE 6,2
      DO 13 I=1,N
 60   READ INPUT TAPE 5,4,(B(J),J=1,LM)
 98   IF(I-KI)12,7,7
 7    KX=KX+1
      KI=KI+K+KX
      ID(1)=ID(1)+2
      ID(3)=ID(3)-1
 12   CONTINUE
      DO 8 J=1, 3
      NX=M-ID(J)
      IF(MD-NX) 8,25,25
 25   A(I,NX)=B(J)
 8    CONTINUE
      DO 9 J=1,NUM
      NX=MD+J
 9    A(I,NX)=B(J+ 3)
      WRITE OUTPUT TAPE 6,200,(A(I,J),J=1,10)
 13   CONTINUE
 200  FORMAT(8F6.2,5X,2F6.2)
C MAKING ELEMENTS ON MAIN DIAGONAL ZERO AND ELEMENTS TO LEFT ZERO
C THIS IS DONE BY ADDING, SUBTRACTING AND MULTIPLYING DIFFERENT ROWS
      WRITE OUTPUT TAPE 6,2
      DO 21 I=1,N
      IF(A(I,M))15,14,15
 14   CALL EXIT
 15   IF(A(I,M) -1.0)16,18,16
 16   STOR=A(I,M)
      DO 17 J=1,LEN
 17   A(I,J)=A(I,J)/STOR
 18   CONTINUE
      DO 21 L=1,M1
      I1=I+L
      IF(I1-N) 19,19,21
 19   I2=M-L
      IF(A(I1,I2))20,21,20
 20   STOR=A(I1,I2)
      DO 41 J=1,MD
```

```
      LM=J+L
      IF(MD-LM) 41,40,40
 40   A(I1,J)=A(I1,J)-STOR*A(I,LM)
 41   CONTINUE
      LM=MD+1
      DO 42 J=LM,LEN
 42   A(I1,J)=A(I1,J)-STOR*A(I,J)
 21   CONTINUE
      DO 91 I=1,N
 91   WRITE OUTPUT TAPE 6,200,(A(I,J),J=1,10)
C MAKING ELEMENTS TO RIGHT OF MAIN DIAGONAL ZERO
      WRITE OUTPUT TAPE 6,2
      DO 34 I=1,N
      NX=N-I+1
      LD=MD-M
      DO 34 L=1,LD
      I1=NX-L
      IF(I1) 34,34,22
 22   I2=M+L
      IF(A(I1,I2)) 23,34,23
 23   STOR=A(I1,I2)
      DO 24 J=M,MD
      LM=J-L
      IF(LM) 24,24,55
 55   A(I1,J)=A(I1,J)-STOR*A(NX,LM)
 24   CONTINUE
      LM=MD+1
      DO 26 J=LM,LEN
 26   A(I1,J)=A(I1,J)-STOR*A(NX,J)
 34   CONTINUE
      DO 74 K=1,10
 74   WRITE OUTPUT TAPE 6,201,(A(K,J),J=1,10)
 201  FORMAT(8F6.2,6X,2F6.3)
C WRITING OUT SOLUTION
      L=MD
      DO 84 J=1,NUM
      L=L+1
      WRITE OUTPUT TAPE 6,65,J
 65   FORMAT(9H SOLUTION,I4)
 84   WRITE OUTPUT TAPE 6,90,(A(I,L),I=1,N)
 90   FORMAT(1X,4E16.8)
      END
 SAMPLE PROBLEM
        10          2           8           6           3
   -1     0     0
    2.0     5.0     5.0     1.0     1.0
    2.0     7.0     7.0     1.0     1.0
    1.0     6.0    -1.0     1.0     1.0
    3.0    10.0     3.0     0.0     0.5
    4.0    -8.0     2.0     1.0     0.5
    2.0    11.0     5.0     0.0     0.0
    8.0    13.0     0.0     0.0     0.0
   -6.0     9.0     0.0     1.0     1.0
    7.0     8.0     0.0     0.5     1.0
   -7.0     7.0     0.0     0.5     0.5
```

```
..I FINITE DIFFERENCE SOLUTION 920134  PLEASE SAVE OUTPUT
..LOAD FORTRAN EXECUTE
 HIGHEST LOCATION LOADED = 20799. NEXT AVAILABLE LOCATION BELOW COMMON



C DIAGONAL SUBSCRIPTED MATRIX WITH ELEMENTS IN PROPER POSITION
 SAMPLE PROBLEM
   0.00   0.00   0.00   0.00   0.00   5.00   2.00   0.00          1.00   1.00
   0.00   0.00   0.00   0.00   0.00   7.00   2.00   0.00          1.00   1.00
   0.00   0.00   0.00   0.00   1.00   6.00  -1.00   0.00          1.00   1.00
   0.00   0.00   0.00   0.00   3.00  10.00   3.00   0.00          0.00    .50
   0.00   0.00   0.00   0.00   4.00  -8.00   2.00   0.00          1.00    .50
   0.00   0.00   2.00   0.00   0.00  11.00   0.00   5.00          0.00   0.00
   0.00   0.00   8.00   0.00   0.00  13.00   0.00   0.00          0.00   0.00
   0.00   0.00  -6.00   0.00   0.00   9.00   0.00   0.00          1.00   1.00
   0.00   0.00   7.00   0.00   0.00   8.00   0.00   0.00           .50   1.00
  -7.00   0.00   0.00   0.00   0.00   7.00   0.00   0.00           .50    .50



C MATRIX AFTER MAKING MAIN DIAGONAL ONE AND ELEMENTS TO LEFT ZERO
 SAMPLE PROBLEM
   0.00   0.00   0.00   0.00   0.00   1.00    .40   0.00           .20    .20
   0.00   0.00   0.00   0.00   0.00   1.00    .28   0.00           .14    .14
   0.00   0.00   0.00   0.00   0.00   1.00   -.17   0.00           .15    .15
   0.00   0.00   0.00   0.00   0.00   1.00    .28   0.00          -.04   0.00
   0.00   0.00   0.00   0.00   0.00   1.00   -.21   0.00          -.12   -.05
   0.00   0.00   0.00   0.00   0.00   1.00   0.00    .45          -.02   -.02
   0.00   0.00   0.00   0.00   0.00   1.00    .01   0.00          0.00   -.01
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00           .02    .06
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00           .09    .17
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00          -.06   0.00



C MATRIX AFTER MAKING ELEMENTS TO RIGHT OF DIAGONAL ZERO
 SAMPLE PROBLEM
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00          .159   .160
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00          .100   .098
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00          .149   .154
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00         -.003   .023
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00         -.136  -.065
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00         -.036  -.058
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00          .002  -.014
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00          .020   .067
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00          .094   .176
   0.00   0.00   0.00   0.00   0.00   1.00   0.00   0.00         -.064   .005



 SAMPLE PROBLEM
 SOLUTION    1
    .15992068E+00    .10019830E+00    .14930597E+00   -.39659100E-02
   -.13608627E+00   -.36413325E-01    .24405600E-02    .20386936E-01
    .94361658E-01   -.64657697E-01
 SOLUTION    2
    .16046822E+00    .98829470E-01    .15409686E+00    .23410679E-01
   -.65465795E-01   -.58684540E-01   -.14406570E-01    .67467252E-01
    .17634896E+00    .59627765E-02
```

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